BASAL AREA AND POINT-SAMPLING

Interpretation and Application

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(Revised)

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By
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ACKNOWLEDGMENTS

Management of the major timber types in the Lake States has been intensified by the application of the basal area method of regulating stocking. Much of the credit for promoting this method should be given to Carl Arbogast, Jr. (deceased), formerly of Marquette, Michigan and to Robert E. Buckman of Washington, D.C., for their research with the U.S. Forest Service in northern hardwoods and pine, respectively. These men have been instrumental in stimulating the authors' interest in the application of basal area and point-sampling concepts.

(This bulletin is a revision by the same authors of Technical Bulletin Number 23 published by the Wisconsin Conservation Department in 1961. Hovind is Assistant Director, Bureau of Forest Management, and Rieck is Director, Bureau of Fire Control.)

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INTRODUCTION

The management of forest stands dictates that the forester have some reliable measure of expressing stocking. Stocking is described by many methods such as percentage of cover, volume per acre, stems per acre and basal area per acre. With some of these methods, stocking levels have not always been clearly or accurately described.

Research substantiates the fact that basal area is a very reliable measure of stocking and is readily understood. It can be accurately measured and is, for all practical purposes, independent of site.

The conventional methods used heretofore to determine basal area were the 100 percent tally of a known acreage or systems of sample plots. Since these methods are laborious and time consuming, the practice of regulating stocking by basal area was seldom applied until recently.

Bitterlich (1948) in Europe described a new but very rapid method of determining basal area per acre by point-sampling. Grosenbaugh (1952a) and others have further translated and expanded on this method and made it applicable to forestry in North America. Since the introduction of this method of determining basal area, it has gained unusually wide acceptance and usage.

This publication attempts to interpret, digest and put together much of the information previously published on basal area and point-sampling. Emphasis is placed upon the practical use of point-sampling in regulating stand density by basal area in Wisconsin and surrounding areas.

WHAT IS BASAL AREA

Basal area as applied to tree measurement is the area in square feet of the cross section of a tree at breast height. Basal area per acre is the total area in square feet of the cross sections of all trees on an acre.

The area of a circle or a tree cross section is expressed by the formula:

$$A = \pi r^2 \text{ or } \frac{\pi D^2}{4} \text{ or } .7854D^2$$

To obtain the area in terms of square feet but still keep the diameter in inches, $.7854D^2$ is divided by 144 giving the formula A (sq. ft.) = $.00545D^2$. Computations in the field, however, are unnecessary as tables are available that give basal area values for each diameter class (Table 1).

The basal area of a tree (a) as compared with one twice its size (A) is as d^2 is to D^2. The areas vary directly with the squares of their diameters and a tree having twice the diameter of another will have four times the basal area. For this reason tree diameters cannot be averaged to determine average tree size. Only by squaring the diameters or averaging basal areas can an accurate average tree size (diameter) of a number of trees be determined. The resulting variation between averaging stem diameters and basal areas is greatest where a wide range of diameters is involved and of less significance where the range is narrow.

Basal area per acre varies with species, timber type and age. Recommended residual stocking levels for pole and sawtimber stands in the Lake States, for example, may run from 70–120 square feet per acre. Unmanaged or virgin forests may possibly run up to 300 square feet particularly in dense northern white pine and hemlock stands or coniferous swamps. With some western species, the basal area per acre may possibly run over 500 square feet. In fact, one redwood 12 feet in diameter has as many square feet of basal area as an average acre of pole-size red pine.

Basal area in a sapling stand is naturally low but increases rapidly through the small pole stage and then gradually levels off as a stand reaches 30–50 years. The culmination of mean annual growth in red pine for instance is about at age 25. In a young red pine stand the annual basal area growth may be 6–7 square feet or more per acre, but as the stand becomes older the annual basal area growth levels off at about 2–3 square feet. Therefore, basal area is a more stable measure of stand density in older stands (30 years and older) than it is in vigorously growing immature stands.

It has been claimed that height growth in general is not greatly affected by stand density. It has also been determined in recent years that there is not as strong a relationship as originally supposed between basal area and site particularly in the Lake States. The main reason for the difference in volume on good sites as against poor sites is height. For all practical purposes, therefore, one can assume that basal area does not vary with site excepting at the extremes.

MEASURING BASAL AREA

One way to determine the basal area per acre of a particular timber stand is to measure all trees on the area, compute the basal area for each diameter class, total these and divide by the number of acres. Most of the time, however, a system of sample plots such as an acre, 1/2 acre, 1/5 acre, etc., has been used to determine the basal area per acre.

The cumulative 1/5-acre tally sheet used in Wisconsin (Fig. 12) has been a convenient form for determining the basal area per acre by 1/5-acre plots. The basal area figures on this form are on a per acre basis as is volume and need no further expansion. A cumulative 1/10-acre basal area tally sheet (Fig. 13) is also used particularly in smaller timber for making a rapid determination of basal area per acre from 1/10-acre plots. Various other tables and those in the U.S. Forest Service Timber Management Field Book, Region 9, are helpful in making quick basal area determinations.

POINT-SAMPLING METHOD OF MEASURING BASAL AREA

A more rapid but yet accurate method of sampling tree basal area and also for obtaining tree volumes was devised by Bitterlich (1948). This method does not require measurement of sample plot areas nor does it require measurement of tree diameters. It is called the Bitterlich system, but is also commonly referred to as point-sampling, variable plot sampling, plotless cruising or plotless timber estimating. Reference to this method will be called point-sampling in this publication.

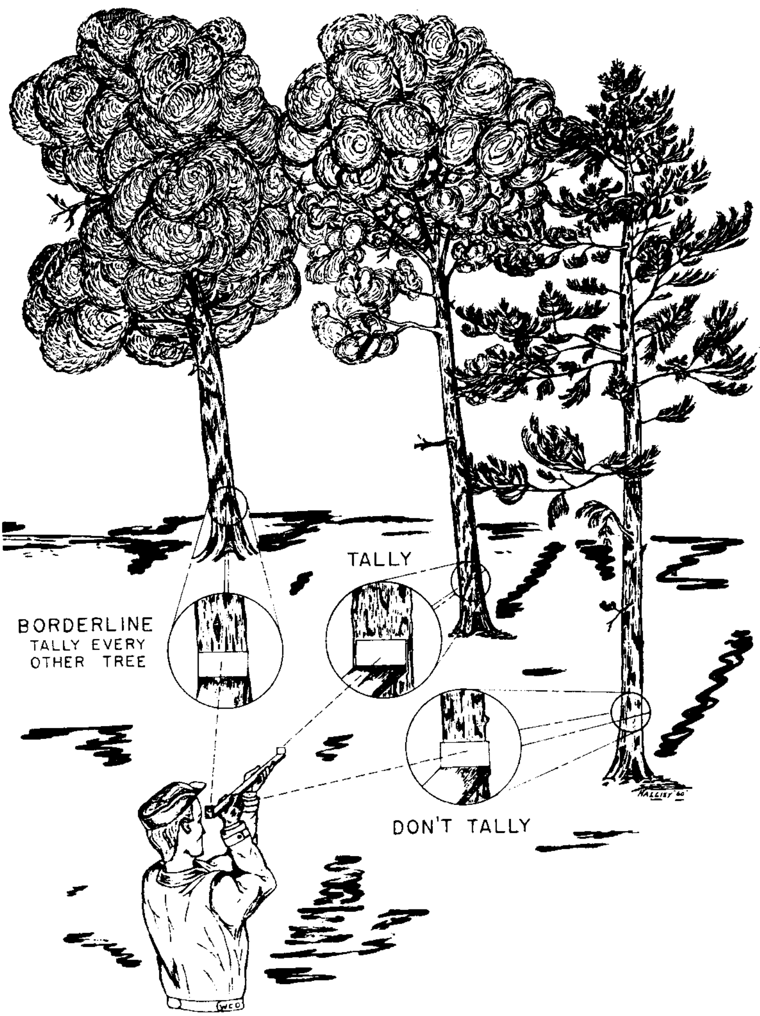


FIGURE 1. Point-sampling with angle-gauge.

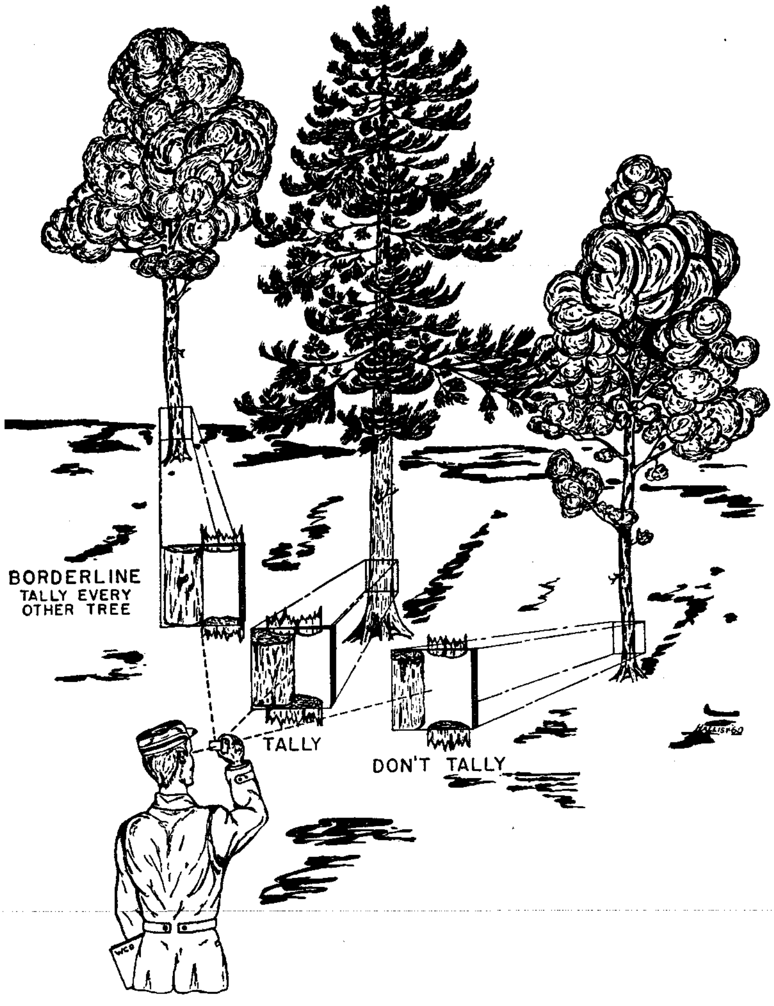


FIGURE 2. Point-sampling with wedge prism.

The basic instruments used in the point-sampling method are the angle-gauge and wedge prism. A cruiser counts the number of trees around a sampling point whose *diameters at breast height* appear larger than the cross-arm of the angle-gauge (Fig. 1). Using a wedge prism, he counts the number of trees whose stem sections at breast height when viewed through the prism do not appear to be detached from the main stem (Fig. 2). If the tree count is multiplied by a predetermined factor (basal area factor—BAF) the basal area per acre is obtained at that particular sampling point. There is a definite correlation between tree basal area and tree volumes, therefore timber estimates may be obtained when counted trees are tallied by merchantable height classes or when the total number of logs or pulpwood bolts in the counted trees are recorded. The number of points to be taken to obtain reliable basal area and volume estimates is discussed later.

The predetermined basal area factor can be any convenient factor based upon the size angle that one selects to use for his gauge or by the strength of the wedge prism. Timber size will determine which factor is most useful. In the average sawtimber stand in the Lake States, an instrument with a basal area factor of 10 is recommended. In uniform or dense pole timber stands the 10 factor is also recommended. On the other hand, if one is working with large sawtimber, a 20-factor or larger might be the most convenient. In light density pole stands a 5-factor instrument may prove to be the most satisfactory since it will reach out farther for the small trees. The use of an instrument having an unwieldy basal area factor can lead to a waste of time, confusion in ascertaining "in" and "out" trees, and unreliable estimates.

The main criteria in selecting a specific basal area factor are stand density and the size of the trees normally involved. In dense pole timber stands some trees are obscured by others, and in large timber the larger trees may be a great distance from the point center, resulting in some trees being overlooked and not counted. For example, using an instrument with a basal area factor of 10, a 36-inch tree is counted up to a distance of 99 feet from the center. One can readily see that such trees can be easily missed in areas where there is any amount of underbrush.

Instruments Used

Basically there are two types of instruments used in point-sampling. One is the *horizontal angle-gauge* and the other the *wedge prism.* Angle-gauges are nothing new to the forester; for example, the hypsometer for measuring tree heights is a vertical angle-gauge, and the Biltmore stick used in measuring tree diameters is a horizontal angle-gauge. Both of these instruments are based upon geometric principles centuries old.

a. A simple **stick-type angle-gauge** with a basal area factor of 10, for instance, can be constructed by mounting a one-inch crossarm at the end of a 33-inch stick. A peephole sight may be mounted at the other end. The angle created by the eye (as the vertex) at the peephole and the outer edges of the crossarm is 104.18 minutes. The ratio of the width of crossarm to the length of stick is 1:33. Likewise the ratio of the tree diameter of a tree just covered by the crossarm and the distance from tree to observer is 1:33 (an example of similar triangles.) In other words, the crossarm will exactly cover a 12-inch (1-foot) tree at 33 feet, a 24-inch (2-foot) tree at 66 feet, etc. (Fig. 3). This ratio of 1:33 gives the instrument a basal area factor of 10 meaning that each counted tree then represents 10 square feet of basal area.

The crossarm and stick of an angle-gauge having a basal area factor of 10 can be of any size as long as this ratio of 1:33 is maintained. The width of

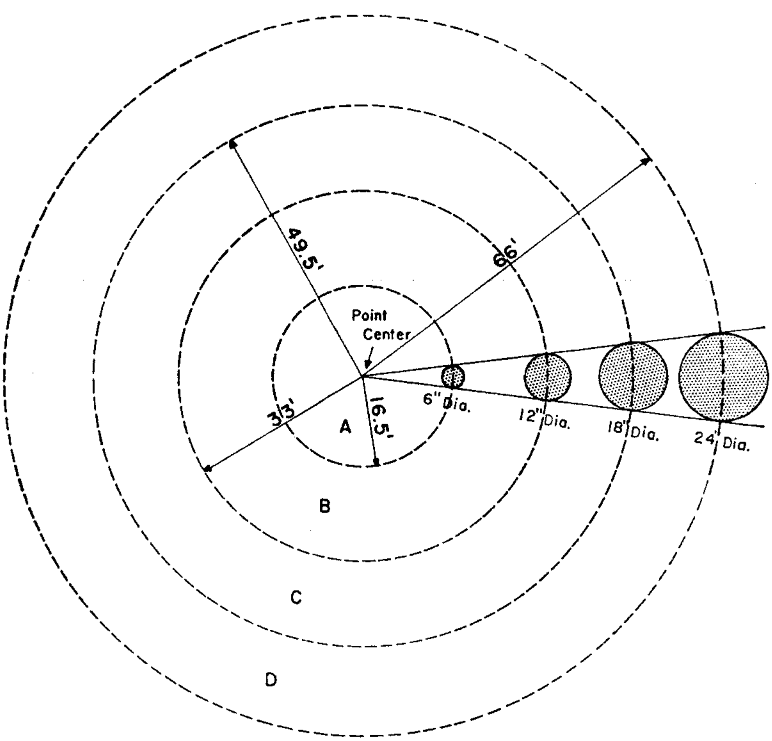
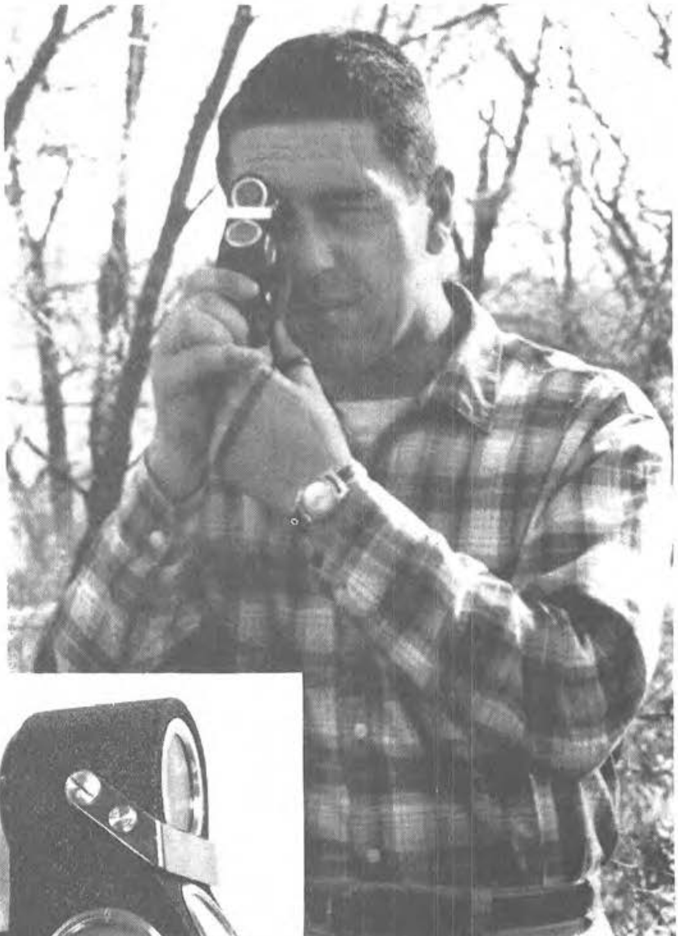


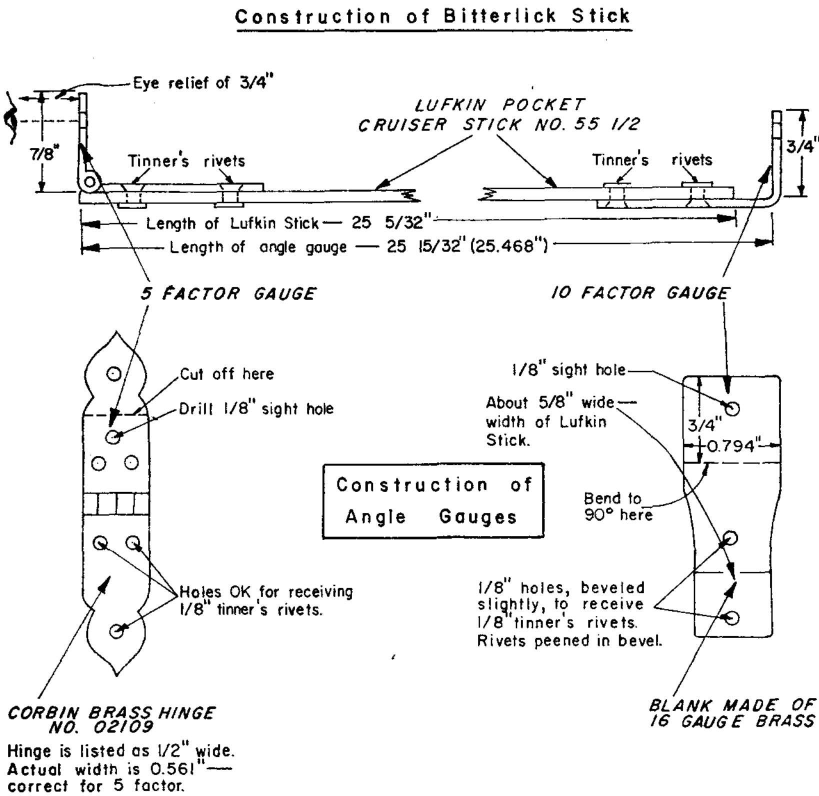
FIGURE 3. Relationship between tree diameter and "plot" distance. BAF 10.

the crossarm per inch of stick is obtained by dividing 1 by 33, or .0303 inches. For a 20-inch stick, for example, the crossarm would have to be .0303 inches × 20 or .606 inches. A convenient, all-around gauge can be made by mounting hinges on a folding hypsometer (Fig. 4). A gauge can be constructed of any length and for any predetermined basal area factor. However, gauges less than 20 inches in length are not recommended because of the difficulty of focusing on a far object and a near object at the same time. The image of the crossarm against the tree becomes blurred as the stick is reduced in length below 16–20 inches. Adjustable or combination angle-gauges can be constructed for two or more factors (Tables 2 and 3). Allowances must be made for the distance from the eye to the end of the stick in figuring the effective length of the gauge (Fig. 4).

b. The **Spiegel–Relaskop** is another of the horizontal-type angle-gauges and also measures tree heights, diameters, distance and slope. It has four built-in angle-gauges and is the most accurate instrument that can be used for determining basal area since it automatically corrects for slope. The cost of this instrument will probably limit its being brought into common usage for the present.



The Spiegel Relaskop and a demonstration of its use in the field.



Computations for Gauge Widths

Total length of Bitterlick Stick	25.468"
Eye relief displacement	+0.750
Effective length of Bitterlick Stick	26.218

	Effective length	X	Width of gauge per 1" of stick	=	Gauge width
10 FACTOR WIDTH	26.218	X	0.0303	=	0.7944"
5 FACTOR WIDTH	26.218	X	0.0214	=	0.5611"

f-i Tomahawk 1962

FIGURE 4. Construction of angle gauges.

c. The **Panama Angle-Gauge** is also a horizontal angle-gauge. This instrument is a short tube with a peephole on one end and a modified crossarm on the other (Fig. 5).

d. An **angle-gauge** can be improvised using coins, thumbnails, sticks, etc., when needed in an emergency. However, one must know the correct width of the substitute crossarm and the reach in order to determine the basal area factor. (See Table 4 for formulae.)

e. A **wedge prism** is a wedge-shaped piece of optically ground glass.

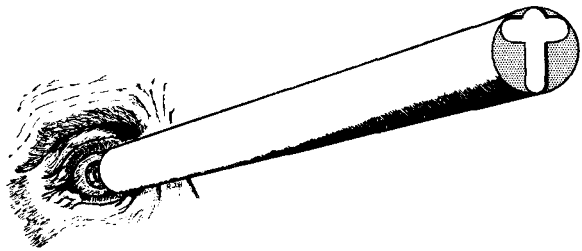


FIGURE 5. Panama angle-gauge.

The principle of the wedge prism as used in point-sampling is based upon the deflection or displacement of light rays. Prism strengths are measured in diopters. A prism of 1 diopter will displace an object 1 unit in 100 units. A 3-diopter prism therefore displaces an object 3 feet in 100 feet.

Three-diopter prisms are commonly used because they have a basal area factor very close to 10 and are reasonable in cost. If ground exactly, 3-diopter prisms would establish a ratio of 1:33⅓ rather than 1:33 and have a basal area factor of 9.8 rather than 10. For a prism to have an exact basal area factor of 10 the prism must have a diopter strength of 3.03. It is possible when purchasing a number of inexpensive 3-diopter prisms to find some having a diopter strength of 3.03.

A wedge prism with an exact basal area factor of 10 has an angle of deflection that is equal to the angle of incidence of an angle-gauge with the same basal area factor. This angle is commonly referred to as the *critical angle* (Fig. 6).

Theory of Point-Sampling

Anyone being initiated to this new method of determining basal area without measuring tree diameters or plot distances may be puzzled as to how it

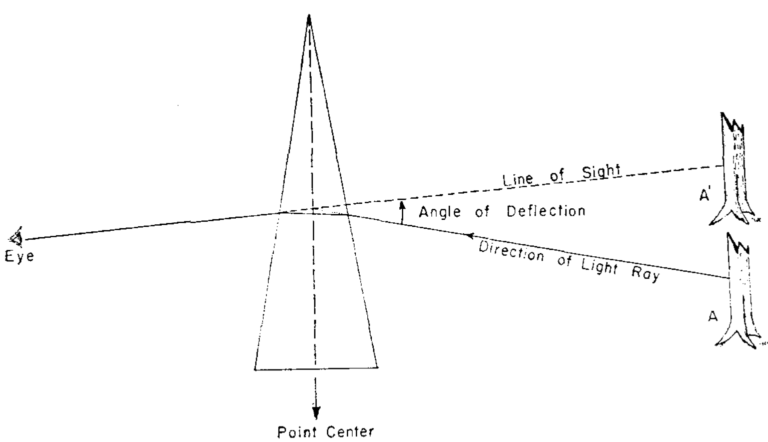


FIGURE 6. Principle of wedge prism. (Object A appears as if at A'.)

works. Several explanations of the theory are offered here to help in understanding the point-sampling concept. A firm grasp of the theory, however, can only be had by working on the ground with the instruments and also by acquiring a thorough understanding of the formulae and tables.

Following are three explanations of the theory based upon three different approaches. All following explanations and sketches apply to an angle of 104.18 minutes which gives the resultant basal area factor of 10. The principles of the theory, however, apply to any angle and its respective basal area factor.

Explanation No. 1

The angle-gauge or prism establishes a ratio between the diameter of a tree that is just on the edge of the "plot"* (and is counted) and the distance from the center of the tree to the point center. With a basal area factor of 10 as mentioned above, the ratio is 1:33. This ratio exists for all tree diameters and is expressed as the fraction, diameter of tree in feet over distance from center of tree to point center in feet (Fig. 3).

A ratio also exists between the area in square feet of a tree that is just in and the area of the circle whose radius is the distance from the tree to the point center. The area of the large circle is 4356 times the area of the tree and this holds for all tree diameters and their respective "plot" areas. Since there are 43,560 square feet in an acre, each counted tree therefore represents 1/4356 of an acre or 10 square feet. Recall that the areas of circles vary with the squares of their diameters. As an example: Assume a tree 1 foot in diameter and just in at 33 feet. Then their respective areas vary as d^2 to D^2. Therefore:

$$\frac{d^2 \text{ (tree diam.)}}{D^2 \text{ (plot diam.)}} = \frac{1^2}{66^2} = \frac{1}{4{,}356}$$

Explanation No. 2

Assume that all trees in the forest are encircled with imaginary rings or zones whose diameters are 66 times the diameter of the respective trees (Grosenbaugh, 1955). Naturally the larger the tree, the larger this ring. In point-sampling, the point center would have to fall within these rings in order for such trees to be counted (Fig. 7). The angle-gauge or prism determines if the point center falls within these rings.

The probability of this point center falling within the ring is proportional to the size of the tree. A tree twice the diameter of another would have four times the probability of being counted as would the smaller. This provides a good sampling method where larger trees which are usually more variable in volume and quality are sampled in greater proportion than the smaller trees which are less variable and of less importance. It is a drastic change

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* "Plot" or "plot" area means the area represented by each tree diameter and does not refer to a fixed acreage.

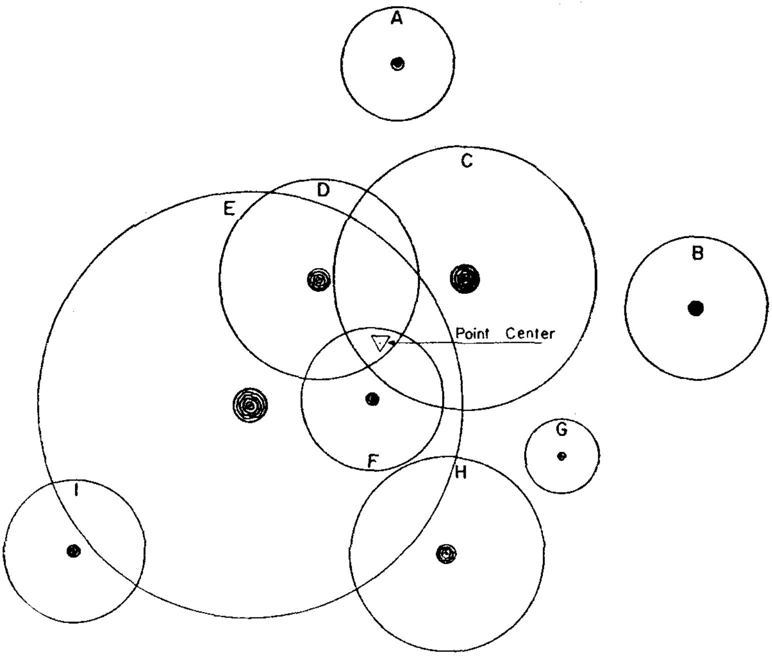


FIGURE 7. Probability of a tree being counted. (C, D, E and F trees are counted.)

from conventional methods where trees are sampled in proportion to their distribution in the stand.

Each counted tree, while representing 10 square feet of basal area per acre, also represents a certain part of an acre and a certain number of trees per acre. A small tree will represent a small area but a large number of trees. For example, an 8-inch tree with a basal area of .349 square feet is on a "plot" with an area of .034889 acres. It takes 28.65 "plots" of this size to make an acre. Each 8-inch tree therefore represents 28.65 trees on an acre; 28.65 $\times$ .349 equals 10 square feet of basal area. At the other extreme, a 42-inch tree represents a "plot" of approximately one acre. A tree this size has about 10 square feet of basal area, and, 10 $\times$ 1 equals 10. Again it should be pointed out that each counted tree irrespective of its size represents 10 square feet of basal area per acre.

Explanation No. 3

Further explanation is offered from a mathematical standpoint (Afanasiev, 1957). The basal area of a tree $= \pi r^2 = \frac{\pi D^2}{4} = .785D^2$ square inches or $\frac{.785D^2}{144} = .00545D^2$ square feet. The size of the "plot" within which trees with the Diameter D are counted $= \pi r^2$. R of the "plot" is 33 times as large as D, hence:

$$\text{Plot area (sq. in.)} = \pi \times (33 \times D)^2$$

$$\text{Plot area (sq. ft.)} = \frac{\pi \times (33 \times D)^2}{144} = 23.7042D^2$$

$$\text{Plot area (acres)} = \frac{23.7042D^2}{43,560} = .000545D^2$$

From the above it is evident that the ratio of the basal area of one tree to the size "plot" (acres) is 10 to 1 ($.00545D^2{:}.000545D^2$). This relationship holds for a tree of any size.

The basal area per acre represented by one counted tree would be:

$$\frac{1 \text{ Acre}}{\text{Plot Area}} = \frac{\text{basal area per acre}}{\text{basal area of tree}}$$

$$\frac{1}{.000545D^2} = \frac{\text{b.a. per acre}}{.00545D^2}$$

$$\text{b.a. per acre} = \frac{1 \times .00545D^2}{.000545D^2} = 10$$

A counted tree regardless of its size or position within the "plot" represents 10 square feet of basal area per acre when using an instrument with a basal area factor of 10. In addition, each counted tree regardless of its position within the "plot" represents a given number of trees per acre and a certain part of an acre depending upon its size. In all cases the actual basal area of a counted tree when multiplied by the number of trees it represents per acre will always equal 10 (Table 5).

A cruiser taking a point in large timber where all trees are *well within* the limits of the "plot" may think that the basal area is more than he reads with his angle-gauge or prism. It must be remembered, however, that the gauge or prism is giving a *per acre* figure and not a figure for the area around the point. For this reason no one should ever attempt to compare a point with a fixed plot. *A point has no area*—trees on the "plot", however, represent certain areas. For example: assume a point with 10 large pine trees each about 52 inches in diameter. The gauge gives a reading of 100 square feet yet the 10 trees actually total about 150 square feet in basal area. *But,* a 52-inch tree represents a "plot" or zone area of 1.5 acres, therefore reducing the basal area to an acre basis, or $\frac{150}{1.5} = 100$ square feet per acre (Fig. 8).

Correct Use of Instruments

Angle-Gauge

With the eye as the point center, the cruiser counts all trees whose diameters at breast height appear larger than the crossarm. Where the trees appear the same size as the crossarm one can either count every other tree or measure the distance to the tree, measure its diameter and multiply the

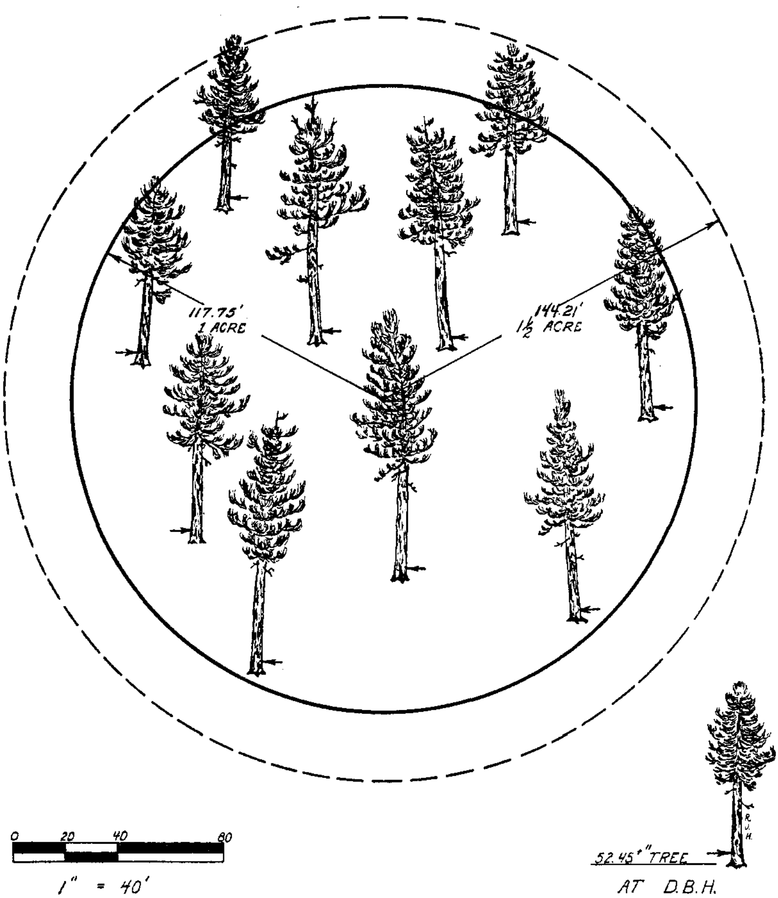


FIGURE 8. Tallied trees are on one acre, but "plot" area is 1.5 acres.

diameter (inches) by a plot radius factor (2.75 for a 10 BAF). If the product is larger than the distance, the tree is counted. Each counted tree is multiplied by the basal area factor to give the basal area per acre. With trees that lean to the right or left of the line of sight, turn the angle-gauge until the crossarm is at right angles to the stem. Trees that lean toward or away from the observer can generally be handled like normal trees.

Trees that are forked above breast height should be counted as one tree for basal area. Trees forked below breast height should be counted as two trees. Be sure to maintain the eye as the point center when making the tree count. It is important to hold the gauge the same distance from the eye as allowed for in determining length of stick. Errors of up to 5 percent or more can result from improper positioning of the gauge.

One must be certain of tallying all trees and especially those larger trees that may be hidden at some distance from the center. Since the cruiser counts trees from a fixed spot, he must make sure not to count dead ones or miss any that may be hidden by other trees. Care must be exercised to maintain the same distance from a hidden tree to point center when necessary to move off point center to view the hidden tree. Where only merchantable trees are to be tallied, one can select a small sapling as a pivot point to be certain of maintaining the point center. However, where the total basal area is to be tallied, selecting a small sapling as plot center should be avoided as this tree will automatically give 10 square feet (BAF 10) whereas under ordinary circumstances few trees this size may actually be counted.

In hilly terrain allowances must be made for slopes that exceed 15 percent (Table 6). Although slope correction tables are available, actually a separate correction factor would have to be applied to almost every tree on the "plot". The only cases where constant slope correction factors for all trees can be applied would be at the bottom of a perfect bowl or the top of a perfect knob (cone). Therefore, in hilly country the Spiegel–Relaskop would appear to be the best instrument to use since it automatically corrects for slope.

Prism

The prism is held at any convenient distance from the eye, but the center of the prism is the point center rather than the eye since the angle begins at the prism. In practice a spot directly beneath the prism becomes the plot center. The observer counts all trees whose stem sections at breast height when viewed through the prism do not appear to be detached from the main stem. Borderline trees are handled similarly to those described in the angle-gauge discussion. Likewise, each counted tree is multiplied by the basal area factor to give the basal area per acre.

Great errors can arise from the improper positioning of the prism. Correct use requires that it be held in a vertical position and at right angles to the line of sight (Fig. 9a). On level ground the top edge should be horizontal. With the top edge held horizontal, rotate prism slightly to determine position of minimum deflection. At this position it can be assumed that the prism is being held at right angles to the line of sight.

A horizontal rotation and/or a dipping of the prism toward or away from the observer increases the diopter strength because of the difference in glass thickness (Fig. 9b). Any rotation within a vertical plane decreases the diopter strength (Fig. 9c). Deviations of 5 degrees in positioning the prism will cause no measurable errors, *but greater deviations can introduce very significant errors.* Extreme carelessness in positioning the prism can change the resultant strength of a 3-diopter prism by nearly 1 diopter.

Other precautions used with the gauge such as hidden trees, forked trees and maintaining the point center also apply. In viewing trees that lean to right or left, tip the prism so that the top is perpendicular to the bole of the tree. Correction for slope can be made by tipping the top edge to the same angle as the slope. In this manner the prism automatically corrects for slope distance (Fig. 10). For this reason and the ease of carrying, the prism

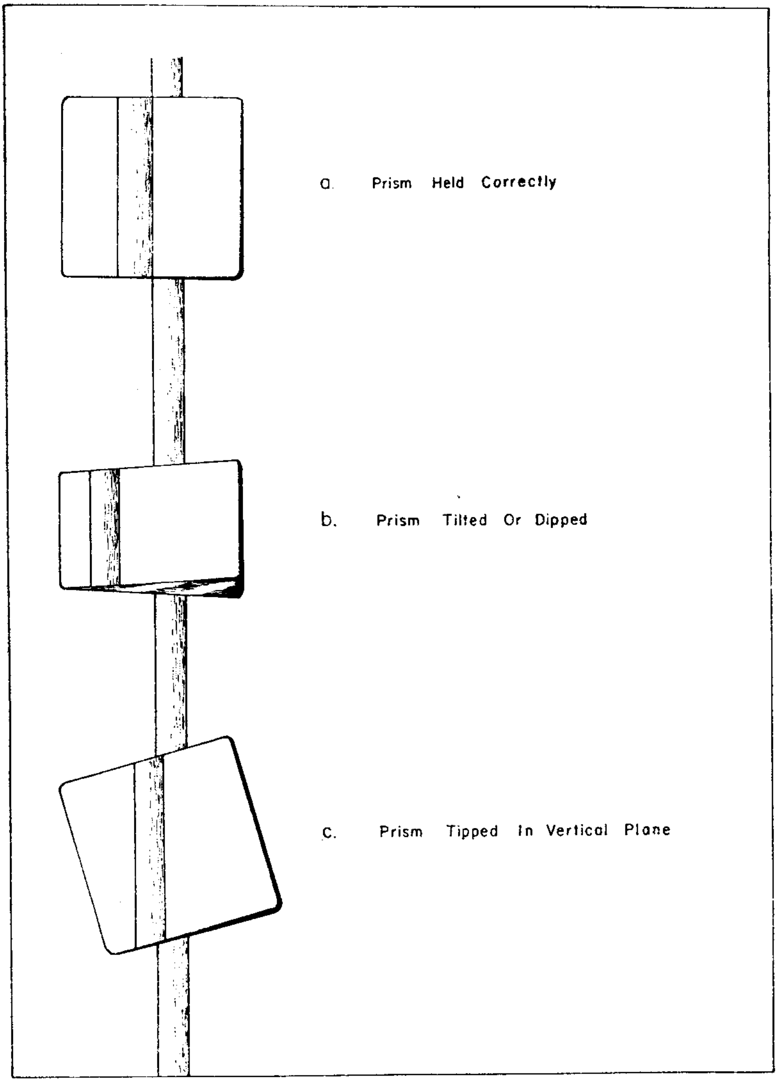


FIGURE 9. Positioning of prism.

has assumed considerable popularity. Careful use of the prism in dense woods is necessary because of the difficulty of maintaining individual tree identity. Most persons, however, will find that the angle-gauge is probably more reliable since there is less chance to err. Various devices have been made to hold the prism level, perpendicular and at right angles to the observer's eye so as to cut down human error. These devices have been frequently described in the Journal of Forestry and various other forestry publications.

FIGURE 10. Correcting for slope.

Practical Tips on Use of Instruments

Through field use a number of handy tips have evolved. A few of these are:

1. With borderline trees, use a Bitterlich tape. Measure the tree diameters to correlate this with the Bitterlich tape measurement. If the tree diameters are equal to or larger than the readings on the Bitterlich tape, the trees must be counted—if diameters are smaller, then the trees are not counted. A Bitterlich tape can be easily made by printing full inch diameters at 2.75 foot (BAF 10) intervals on the reverse side of a cloth tape.

2. In measuring trees to determine if they are "in" or "out," they should be calipered perpendicular to the line of sight.

3. Always attempt to start the tree count from the same cardinal direction so as to be consistent.

4. Make a double-check of "plot" before leaving it.

5. Where possible, use devices such as a Purdue point-sampling block for holding prisms steady.

6. Darken the crossarm on an angle-gauge for better contrast.

7. Mark the basal area factor on the crossarm where using two or more different basal area factors.

8. Consider a penny as an emergency gauge. The "crossarm" is ¾ inch, and if held at 24¾ inches, the penny gives a basal area factor of 10.

9. When sighting with the angle-gauge, line one side of the crossarm with one side of a tree. If bark appears on the other side of the crossarm, the tree should be counted.

10. Rest the gauge on some object such as a Jacob staff to obtain greater accuracy.

11. Adjust eye shade properly in order to obtain clearer readings with the Spiegel–Relaskop.

Determining the Basal Area Factor

If an angle-gauge is carefully constructed, there should be no need for checking it for the exact basal area factor. It would be a good idea, however, to do so as an added precaution.

Prisms, and particularly the inexpensive ones, may not be ground to the exact diopter desired. The more expensive prisms are guaranteed to have a maximum tolerance of one minute from the specified angle of 104.18 minutes required for a 3.03-diopter prism with a BAF of 10. This means a 3.03-diopter glass could have a BAF ranging from 9.8 to 10.2 and a possible error in the field of ± 2 percent (Bower et al., 1959).

Greater variations of course will be found in the inexpensive prisms where the diopter strength may vary by 0.1 diopter more or less. Because of this, one cannot assume that a 3-diopter prism has a BAF of 9.8 and a 2-diopter one of 4.356. An inexpensive prism could be off 10 percent or more even before human errors are introduced. However, it might be desirable and cheaper in the long run to buy a number of inexpensive prisms and select for use only the more exact ones.

All prisms should be calibrated to determine the exact basal area factor and this figure etched on the glass or scratched on the pouch. Several calibrations should be made for each prism to arrive at the exact factor. Even with a prism ground to the exact diopter or BAF, it is necessary for each individual using that prism to calibrate it since eyesights vary. The calibration is done as follows:

1. Place a rectangular target of exactly 1 foot (or any carefully measured width) vertically against a contrasting background. Do not use a tree for this calibration unless its diameter has been calipered
2. Back away from the target, and viewing the target at eye level through the prism, exactly line up the right side of displaced portion with the left side of the target, with prism held in right hand and thin edge to left (Fig. 11).

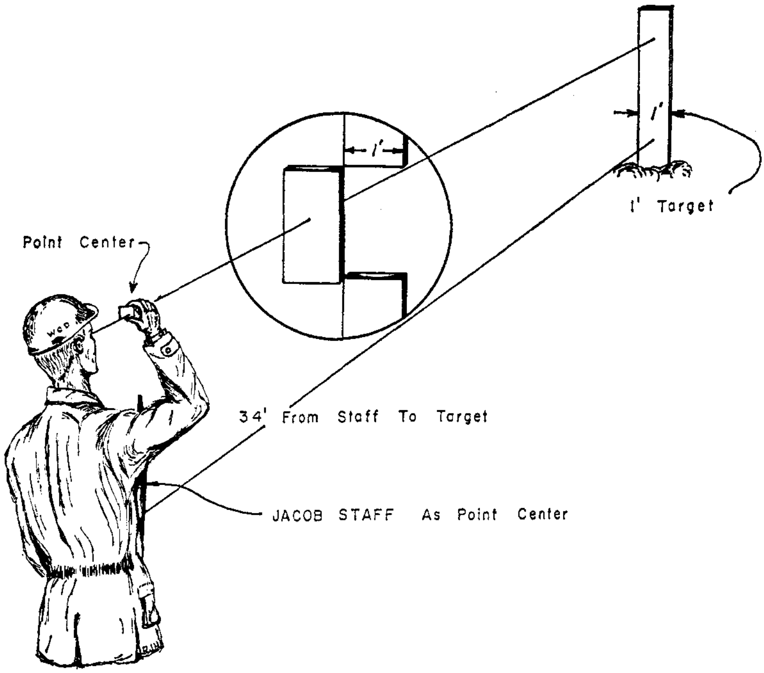


FIGURE 11. Calibration of prism. BAF 9.4.

3. Measure distance in feet from target to prism.
4. Using formula below, compute BAF.

$$BAF = 10{,}890 \left(\frac{w}{d}\right)^2$$

w = width of target in feet
d = distance from target to prism in feet

The same procedure and the same formula can be used to calibrate an angle-gauge. With an angle-gauge the edges of the crossarm must line up with the edges of the target.

Without any magnification of the instruments, there can be significant differences in the calibration of basal area factors between individuals. The main reason for this, of course, is in making decisions on borderline trees. Therefore, with gauges having no magnification it would seem possible that calibrated basal area factors can be rounded slightly for field use—for example, 9.9 to 10.

When the basal area factor has been determined for a particular gauge or prism, each counted tree will represent a number of square feet equal to the calibrated basal area factor. Where an odd BAF is encountered, a handy reference table should be prepared showing various tree number and basal area relationships to simplify field computations.

Once basal area factors have been determined, the relationship of volume between them is directly proportional, and the relationship of area is inversely proportional. Up to this point computations and relationships are largely based on squares or square roots. Recall how areas of circles vary with the squares of their diameters.

USE OF BASAL AREA AND POINT-SAMPLING IN FOREST MANAGEMENT

As a Measure of Stocking

Basal area has come into the limelight in recent years as a measure of stocking levels and consequently as a basis for cutting recommendations and marking guides. In the Lake States, tables of recommended levels of stocking using basal area per acre have been prepared for most timber types. The use of basal area as a level of stocking represents a convenient and unbiased method of determining the intensity of a thinning or harvest cut.

As an example, the following is quoted from Arbogast (1953) and refers to stocking in northern hardwoods:

> "When we started using the selection system we had to talk the language of the logger and mill owner in order to sell the idea that they could afford to cut selectively. It was natural, therefore, that we turned to a unit of measurement with which they were familiar, net board feet per acre, to express stocking level. We soon realized that this would not work. Volume is an expression of site. So if we were going to use volume as a stocking guide, we would also have to include some sort of site index, as the stands varied from 4,000 to 15,000 net board feet per acre. We had no precise expression of such site indexes.
>
> "The next attempt to express stocking was in terms of the percent of the total net merchantable stand to be cut. For example, we spoke of '30 percent cuts' and '50 percent cuts'. It was an improvement because now site was eliminated. Although this method is still used extensively, it has many disadvantages. Some of them are:
>
> (1) It does not take into consideration the condition of the timber. A stand with much cull is actually cut heavier at a given percent than one with little cull. Using gross volume as a base would eliminate this difficulty.
>
> (2) Net board feet measurements are very difficult to make in the forest, because only indirect methods can be used. No matter how carefully the basic measurements are made, large errors are possible because of the nature of the unit.

(3) The relationship between percent of cut and response to release does not hold very well except in localized areas. Twenty-five percent of 4,000 feet represents a lower stocking than 25 percent of 15,000 feet.

(4) The attention is focused on the amount to be cut when it should be on the trees to be left.

"The use of basal area per acre as a unit of stocking now appears to eliminate many of the disadvantages previously mentioned. It can be accurately and simply measured. It is independent of site. It can include all the size classes in the stand, not just those sizes that are merchantable for sawlogs. And, finally, there is very close correlation between residual basal area and volume growth."

Basal area stocking level guides for many of the timber types have been developed which aim at producing optimum growth in quantity and quality. These guides are flexible in that they represent average conditions, so it is possible to vary slightly one way or the other. In plantations, for example, where spacing in uniform, the stand can probably carry a high basal area. A high basal area may have to be carried temporarily in unmanaged dense even-aged stands because of the spindly nature of the trees, and the danger of windthrow and breakage. High basal areas may want to be maintained also for raising special products, such as posts, poles and piling. In general, where cutting in dense stands is done from below and dominants and co-dominants left, the reduction in basal area to the desired level can be made in one cut. Basal areas slightly lower than the recommended levels may be justified when the objective is to increase diameter growth and crown development. Research, however, has shown that cutting below the recommended level can lead to sprouting of hardwoods, development of brush and sod conditions and a decrease in quality of the residual stand.

The objective of management should be to leave a residual basal area of good risk and high quality trees at the optimum level. In types managed as uneven-aged, it is imperative that the recommended residual basal area be distributed among the various size classes (Table 7). While initial cuts aimed at removing poor quality trees may bring lower financial returns, future cuts will more than make up the difference.

As Guides for Marking

Foresters have long recognized the importance of using basal area as a guide in treating timber stands, but due to the laborious methods of obtaining basal area, it was seldom used. With the introduction of the angle-gauge and prism, basal area is now easily obtained and is being used more and more extensively for stand regulation.

Where the purpose is to determine the average basal area per acre, such as in a uniform plantation, a pre-determined route should be followed with the spacing of "plots" at random. In such uniform timber types, up to 10 points may suffice for an average basal area figure for the stand. The mechanics of taking the point have been discussed elsewhere and should be followed in this procedure. The fieldman will tally the total number of trees counted at each point. At the end of his route, he will total all counted (in) trees, divide by the number of points taken and multiply by the basal area factor of his particular instrument to obtain the average basal area per acre.

When determining the basal area per acre in variable timber stands, the location of points is very important. No attempt should be made to average basal area for an entire block of variable timber even though it may be the same timber type and condition class. Where the management is aimed at reducing the stocking of the denser portions of this block, it would be unwise to average in small openings and understocked areas to arrive at a base for making cutting recommendations. Each like portion of the stand should be treated as a unit and within these like portions several points may be averaged both before and after marking. The procedure of taking before and after points is repeated as often as necessary as the markers move through the timber. When the markers become more familiar with recognizing basal area levels, it may be assumed that less before and after points are necessary. Experienced markers after working with point-sampling instruments for some time may be able to estimate basal area levels as well as experienced cruisers can estimate diameters and height. Cruisers, however, do check their measurements frequently and the same should be done by tree markers using the point-sampling method of estimating basal areas.

In training a marking crew or when entering a new area, it may be desirable to lay out sample strips of an acre or more on which all trees are measured and the exact basal area is known. When this strip is marked to the desired level, it serves as an excellent model of what the rest of the stand should look like. It helps the markers to better visualize the job ahead as to spacing and distribution of growing stock. Point samples are taken within this strip to check the instrument readings with the actual basal area. The crew should refer to this strip several times during the marking operation, or additional strips may be established as the job moves along. In stands that do not vary greatly in diameter, tree markers will find the spacing guide (Table 8) very useful in marking to a desired stocking level.

There is controversy as to whether or not it is possible to progressively mark around points. In our opinion, it may be possible to do so in stands where trees are of one size and uniformly distributed. However, in stands such as northern hardwoods where all size classes are present, it would appear to be impractical. "Plot" areas would have to overlap enough so that all trees that are to receive treatment actually fall within one or more "plots". The larger trees determine the "plot" size; therefore one would have to visualize "plot" sizes for each of the smaller diameter classes. The system of laying out sample strips described above would seem to be the most applicable.

In Timber Estimating

Volume Determinations

A board foot–basal area volume table is constructed by first taking a volume table and dividing the volumes by the basal area of their respective diameters. The resultant table will then read board foot volumes per square foot of basal area (Table 9). It is apparent that the volumes per square foot of basal area do not vary appreciably within specific height classes. In other words, the volumes for any diameter for a certain height are consistent and for all practical purposes diameter need not be considered.

The volumes for each height class can be averaged to give the board foot volume per square foot of basal area for the respective height classes. Where a 10-factor gauge or prism is used, this average volume, if multiplied by 10, will give the volume per acre for each tree tallied in its respective height class. Further, if these volumes are divided by the number of 16-foot logs in each height class, a value of approximately 600 board feet (Scribner) per log is obtained. (The figure of 600 works out satisfactorily in the Lake States). In other words, if the number of 16-foot logs in trees within a "plot" are counted and multiplied by 600, the result is the average gross board foot volume per acre. While the average board feet per 16-foot log may be 600 feet, logs in short trees have a slightly higher value and logs in tall trees have a slightly lower value. This is a simplified method of timber estimating and can be used as a rapid check of volume.

The same thing can be done to a cordwood volume table (Table 9), and it is found that each 8-foot stick counted on a "plot" represents approximately 0.6 cord per acre using a BAF of 10. Here again it is found that volumes per square foot of basal area do not vary appreciably within specific height classes.

Where a basal area factor of 5 is used, the gross volumes per log and stick are about 300 board feet and .3 cords, respectively. Volume factors for the various basal area factors are given in Table 10. These volume factors are in direct proportion to the volume factors for the 10 basal area factor.

With cordwood it has been found that the formula

$$\frac{\text{No. of sticks on point} + \text{no. of trees on point}}{2} = \text{volume per acre in cords}$$

(BAF 10). For a basal area factor of 5, the formula would read the same except the divisor is 4; with a BAF of 20, the divisor is 1. These simple formulae work best in stands of average height. They do not work with sawlogs.

For timber cruises, a tally sheet that will record counted trees by height classes is preferred to the use of the average volume figures cited above (600 board feet). Height estimates should be checked frequently as they have a very significant influence on volumes. As a refinement to averaging volumes within a height class, it is suggested that the volumes in each height class be weighted according to the normal distribution of diameters found within the respective height classes. For example, the frequency of ½ log trees is in the smaller diameters, whereas the occurrence of 3 log trees is generally in the larger diameters. This frequency distribution can be obtained locally by the examination of a number of fixed plot tally sheets. Grouping sawtimber for example by small and large is another refinement that can be used so as to give accurate results but yet not requiring tallying by diameter (Fig. 14). On the other hand, trees should be tallied by both height and diameter classes where additional information such as distribution of size classes or other analytical data are needed (Fig. 15).

In certain uniform stands Buckman (1961) has devised formulae for determining stand volumes in cubic feet, cords, and board feet using basal area and the average height of dominants and co-dominants. An example is the cordwood formula where

$$V = \text{Volume}$$
$$B = \text{Basal Area}$$
$$H = \text{Height}$$
$$V \text{ (cords)} = .003958 \; B \; H$$

Fixed Area Estimates

Timber cruising is the art of estimating the volume of timber on a given tract of land. Many methods of cruising have been used, and perhaps there is no one universal system. Some of the standard cruising methods are as follows:

1. 100% cruise—all trees are tallied.
2. Ocular—the timber estimator walks through the tract to be cruised and is able through experience to determine the volume. This method can be, and often is, very inaccurate and should be used only by very experienced timber estimators.
3. Strip method—usually a one-chain-wide strip run several times through a description, depending on the percent of cruise (by area) desired.
4. Line sample plot method—sample plots, usually circular of a given size, are taken along a line at regular intervals in cardinal directions. The number of lines to run depends upon the percent of cruise (by area) desired.
5. Random sample plots
   (a) Stratified—sample plots that are placed at random throughout a given type, size and density class. These plots are allocated on the basis of the types, sizes and density classes within the tract to be cruised.

(b) Unstratified—sample plots that are placed at random throughout the tract to be cruised. The map used in locating the plots will probably contain just the timber types without the size and density classes.

All of these methods can yield acceptable results in the hands of the careful, observant and experienced timber estimator. In cruising, the results will be based on (1) the percentage of cruise by area, and (2) the percentage of accuracy (by volume) desired. In most cases, when a cruiser reports that he has finished a 10 percent cruise, he means that he has cruised *10 percent of the area,* not 10 percent of the volume. Only in very uniform timber of large acreages will the error in volume, expressed as a percentage, and the percent of cruise be equal. *Accurate cruising demands that the number of plots taken be commensurate with the uniformity of the timber and its acreage.* The accuracy to strive for in cruising depends upon the purpose for which the tract is being cruised.

Point-Sampling Estimates Techniques

The use of point-sampling brings in a whole new concept in cruising. Prior to the actual cruise, a reconnaissance of the tract should be made to determine the variability of the timber and the number of points needed. After the number of points are known, they should be located at random on the map sheet or aerial photograph. The cruiser should arrive at the same place on the ground as noted on the map sheet or photograph so as not to introduce bias. He establishes the point center but does not move it because of openings, or dense cover. With a 2-man crew, the compass man becomes the tally man and the cruiser will take his count with the angle-gauge or wedge prism. Prior to this count the cruiser should make a quick reconnaissance of the possible "in" trees and make mental note of cull defect and merchantable height. The estimator then proceeds to determine the "in" trees around the point, starting from a cardinal direction and an outstanding object, and works in a clockwise direction. The estimator should call out the species and log or bolt height, and the tally man records them on a cumulative point-sampling tally sheet. After the estimator is finished with the point, he should do a spot check to see that all the "in" trees are tallied. The tally man should be certain that he correctly notes the number of points sampled.

Tally all trees by log or bolt height and species on the cumulative point-sampling tally sheet (Figs. 14 and 15). Where the tally sheet also includes diameter classes in addition to log or bolt height, these diameters can be estimated from the point center. However, again it must be emphasized that in point-sampling, cruising height measurements and height instruments are of much more importance than diameter measurements and diameter tools.

The compilation of the point-sampling cruise data will be treated as any other sample data; that is, the data must be summarized into volumes per acre and multiplied by the acres in the tract that was cruised. Allowances, of course, must be made for defect and cull.

How many points are needed

With the fixed area estimates, most of the emphasis is based on the acreage cruised. With the point-sampling method, the number of points to be taken is based on the uniformity or variability of the timber to be cruised. Available forestry literature does not attempt to give, in tabular form, the number of points needed for a certain size tract to be cruised. Therefore, at present, there is no way to compare the number of fixed plots needed in a line-plot cruise as against the number of points needed for the same acreage. The main reason for not being able to make this comparison is that plots have a constant size whereas the points have no definite area. However, statistics do provide the following formula in determine the number of points needed:

$$n = \frac{c^2}{e^2}$$

where n = number of points to be taken
c^2 = coefficient of variation squared
e^2 = percent of error squared

In order to determine the coefficient of variation, the following formula is used:

$$c = \frac{100SD}{M}$$

where c = coefficient of variation
SD = standard deviation
M = mean (in volume)

The formula for standard deviation is:

$$SD = \sqrt{\frac{\Sigma\,(x^2)}{N-1}}$$

where SD = standard deviation
x = deviation from the mean
N = number of samples

The formula $n = \frac{c^2}{e^2}$ for one standard error is used where accuracy is quoted as correct two out of three times (66⅔ percent). However greater accuracy may be desired within two standard errors (95 percent) or correct 19 times out of 20. The formula is this case for determining the number of points needed is:

$$n = \frac{4c^2}{e^2}$$

This means that many more points are required but the accuracy level is greatly increased (Table 11).

One unknown for which values must be obtained is the coefficient of variation. The coefficient of variation or the mean volumes can be used in the following ways:

1. In the reconnaissance of a timber type, the observer takes a number of points to determine the volume per acre. This information in turn is plugged into the above formulae.
2. After the reconnaissance the observer should be able to tell the mean volume per acre. When the mean volume is determined then refer to Tables 11 and 12, depending upon the accuracy desired for the number of points needed.
3. If local cruise data for similar tracts are available, these figures can be inserted in the formulae or the mean volumes used with Tables 11 or 12.

It is very important to keep in mind that the number of points shown in the tables or obtained by formulae do not refer to any set acreage.

Strict adherence to the point allocation table (Tables 11 and 12) sometimes may prove to be very impractical. For example: If a 5-acre tract is not uniform in size class and volume, the number of points needed according to the table may be 100 or more. In this case good judgment is required.

Some rules of thumb for determining the number of points needed which may give acceptable results are:

1. In saw-timber stands, use the same number or slightly more points (BAF of 10) as 1/5 acre plots.
2. In pole timber stands, use at least twice as many points (BAF of 10) as 1/5 acre plots.
3. Assume an average tree diameter and determine from Table 5, the "plot" area that such diameter would represent and allocate points accordingly.
4. Never use less than 10 sample points regardless of size of tract or basal area factor used.

These formulae and procedures are also used in determining the number of points needed for basal area estimates. Basal area figures are substituted for volume figures. In general, basal area estimates are likely to be less variable than are volume estimates.

Often it is necessary and desirable to determine the percent of error for a particular cruise. If accuracy is desired at the 2 standard error level (95%), the following formula is used:

$$e = \frac{2c}{\sqrt{n}}$$

where e = percent of error
c = coefficient of variation
n = number of points taken

The acceptable error will depend upon the value of the timber, purpose of the cruise and administrative accountability.

SUMMARY AND CONCLUSIONS

The Bitterlich method of point-sampling was slow in taking hold in this country. A lack of understanding of the basic principles was the main reason for the aversion to its use. Grosenbaugh, Bruce and others have pioneered in explaining the theory and application behind the system. This paper puts together much of the information previously published on basal area and point-sampling, with emphasis upon the practical use of point-sampling in forest management in Wisconsin.

This system dispenses with the need for measuring tree diameters and fixed plot radii for determining basal area and volumes per acre. Instead, point-sampling employs the use of angle-creating instruments which establish definite ratios between tree area and "plot" area. Relationship of any tree size to its respective area is constant for any set instrument. This constant ratio establishes a factor known as the basal area factor. The tree count around a point when multiplied by the basal area factor gives basal area per acre and with supplementary information will give volume per acre. Trees are sampled in respect to their size, with trees of larger size and value being sampled in greater proportion than less valuable smaller trees.

Experience in the use of point-sampling as compared to other methods has instilled confidence in the system. Chief advantages are its speed of application and its accuracy. The efficiency of a field staff is greatly increased because it permits one-man crews to gather management data.

This system is finding great use in forest reconnaissance, marking and timber estimating. Many agencies have replaced fixed sample plot procedures with point-sampling. Even timber surveys are being conducted and others proposed using the point-sampling concept.

Where very accurate information is desired, such as timber estimates, strict adherence must be given to taking the proper number of points. In compartment reconnaissance and in marking, experience will dictate the number of points needed to give satisfactory results.

The point-sampling system is simple to apply. Yet, for reliable information the forester must heed all words of caution on the proper use of the instruments. This system is more than a novelty. Foresters and technicians are urged to use this system to simplify field procedures and increase efficiency.

APPENDIX

(The tables included here are basic. Modifications or adaptations of these tables can be prepared to fit specific field needs.)

TABLE 1

Basal Area Table for Any Number of Trees, by Inch Classes

Diameter	Total Basal Area For Following Number of Trees[1]								
	1	2	3	4	5	6	7	8	9
Inches					**Square feet**				
1	.005	.010	.015	.020	.025	.030	.035	.040	.045
2	.022	.044	.066	.088	.110	.132	.154	.176	.198
3	.049	.098	.147	.196	.245	.294	.343	.392	.441
4	.087	.174	.261	.348	.435	.522	.609	.696	.783
5	.136	.272	.408	.544	.680	.816	.952	1.088	1.224
6	.196	.392	.588	.784	.980	1.176	1.372	1.568	1.764
7	.267	.534	.801	1.068	1.335	1.602	1.869	2.136	2.403
8	.349	.698	1.047	1.396	1.745	2.094	2.443	2.792	3.141
9	.442	.884	1.326	1.768	2.210	2.652	3.094	3.536	3.978
10	.545	1.090	1.635	2.180	2.725	3.270	3.815	4.360	4.905
11	.660	1.320	1.980	2.640	3.300	3.960	4.620	5.280	5.940
12	.785	1.570	2.355	3.140	3.925	4.710	5.495	6.280	7.065
13	.922	1.844	2.766	3.688	4.610	5.532	6.454	7.376	8.298
14	1.069	2.138	3.207	4.276	5.345	6.414	7.483	8.552	9.621
15	1.227	2.454	3.681	4.908	6.135	7.362	8.589	9.816	11.043
16	1.396	2.792	4.188	5.584	6.980	8.376	9.772	11.168	12.564
17	1.576	3.152	4.728	6.304	7.880	9.456	11.032	12.608	14.184
18	1.767	3.534	5.301	7.068	8.835	10.602	12.369	14.136	15.903
19	1.969	3.938	5.907	7.876	9.845	11.814	13.783	15.752	17.721
20	2.182	4.364	6.546	8.728	10.910	13.092	15.274	17.456	19.638
21	2.405	4.810	7.215	9.620	12.025	14.430	16.835	19.240	21.645
22	2.640	5.280	7.920	10.560	13.200	15.840	18.480	21.120	23.760
23	2.885	5.770	8.655	11.540	14.425	17.310	20.195	23.080	25.965
24	3.142	6.284	9.426	12.568	15.710	18.852	21.994	25.136	28.278
25	3.409	6.818	10.227	13.636	17.045	20.454	23.863	27.272	30.681
26	3.69	7.38	11.07	14.76	18.45	22.14	25.83	29.52	33.21
27	3.98	7.96	11.94	15.92	19.90	23.88	27.86	31.84	35.82
28	4.28	8.56	12.84	17.12	21.40	25.68	29.96	34.24	38.52
29	4.59	9.18	13.77	18.36	22.95	27.54	32.13	36.72	41.31
30	4.91	9.82	14.73	19.64	24.55	29.46	34.37	39.28	44.19
31	5.24	10.48	15.72	20.96	26.20				
32	5.59	11.18	16.77	22.36	27.95				
33	5.94	11.88	17.82	23.76	29.70				
34	6.31	12.62	18.93						
35	6.68	13.36	20.04						
36	7.07	14.14	21.21						
37	7.47								
38	7.88								
39	8.30								
40	8.73								

[1] Data are given for one through nine trees only. However, the table can be applied to any number of trees by shifting decimals and adding. Thus, if 129 3-inch trees are found on a plot, the total basal area is found by multiplying the basal area for one 3-inch tree by 100 and adding to it the basal area of two 3-inch trees multiplied by 10, plus the basal area of nine 3-inch trees: (.049) (100)+(.098) (10)+.441=6.321.

TABLE 2

Relationships Between Crossarm and Length of Angle-Gauge

Width of Crossarm (Inches)	Length of Angle-Gauge—Inches		
	BAF 5	BAF 10	BAF 20
.25	11.67	8.25	5.83
.30	14.00	9.90	7.00
.35	16.33	11.55	8.17
.40	18.67	13.20	9.33
.45	21.00	14.85	10.50
.50	23.34	16.50	11.67
.55	25.67	18.15	12.83
.60	28.00	19.80	14.00
.65	30.34	21.45	15.16
.70	32.67	23.10	16.33
.75	35.00	24.75	17.50
.80	37.34	26.40	18.66
.85	39.67	28.05	19.83
.90	42.00	29.70	21.00
.95	44.34	31.35	22.16
1.00	46.67	33.00	23.33

BAF 5 has a ratio of 1/46.67; therefore length of stick is 4.667 inches per .1 inch of crossarm.

BAF 10 and 20 have ratios of 1/33 and 1/23.33; therefore length of stick is 3.300 inches and 2.333 inches per .1 inch of crossarm, respectively.

TABLE 3

Factors for Point-Sampling According to Basal Area Factor and Angle Sizes

(From Grosenbaugh, 1955)

Basal Area Factor	Angle Size		Ratio (Tree diameter to plot radius)	Plot Radius Factor	Cross-arm Length Factor	Calibration Distance Factor
	Minutes	Diopters				
1	32.94	.96	1/104.4	8.696	.0096	104.4
2	46.59	1.36	1/73.8	6.149	.0136	73.8
3	57.06	1.66	1/60.2	5.021	.0166	60.2
4	65.89	1.92	1/52.2	4.348	.0192	52.2
5	73.66	2.14	1/46.7	3.889	.0214	46.7
10	104.18	3.03	1/33	2.75	.0303	33.0
15	127.59	3.71	1/26.9	2.245	.0371	26.9
20	147.34	4.29	1/23.3	1.944	.0429	23.3
25	164.73	4.79	1/20.9	1.739	.0479	20.9
30	180.46	5.25	1/19	1.588	.0525	19.0
35	194.92	5.67	1/17.6	1.470	.0567	17.6
40	208.38	6.07	1/16.5	1.375	.0606	16.5
50	232.99	6.79	1/14.75	1.230	.0678	14.75
60	255.23	7.44	1/13.46	1.123	.0743	13.46

For all practical purposes the size of angle, diopter and crossarm length factor vary directly as the square root of the basal area factors, whereas, the ratio, plot radius factor and calibration distance factor vary inversely as the square root of the basal area factor.

For example:

a. An instrument with a BAF of 20 has twice the angle size of an instrument with a BAF of 5.

b. An instrument with a BAF of 20 has one-half the plot radius factor of an instrument with a BAF of 5.

c. An instrument with a BAF of 10 has an angle equal to the angle of a BAF of $5 \times \sqrt{2}$; an angle of a BAF of 15 equal to angle of BAF of $5 \times \sqrt{3}$; etc.

d. An instrument with a BAF of 10 has a plot radius factor of $\frac{1}{\sqrt{2}}$ x plot radius factor of 5, etc.

TABLE 4

General Formulae Used in Computing Basal Area Factors

1. *Area of circles*

$$A = \pi r^2 = \frac{\pi D^2}{4} = .785D^2$$

$$A \text{ (sq. ft.)} = \frac{.785D^2}{144} = .00545D^2$$

(D = diam. in inches)

The area of circles varies with the squares of their diameters.
A: a as $D^2 : d^2$

Like water pipes, a tree cross section having twice the diameter of another has *four* times the area.

2. *Basal area factors*

$$BAF = 10{,}890 \left(\frac{w}{d}\right)^2$$

(For use in computing BAF for gauge or prism, adapted from ---------- $BAF = \dfrac{43{,}560}{1 + 4\left(\dfrac{d}{w}\right)^2}$

BAF = Basal Area Factor
w = width of target in feet
d = distance in feet to target

$BAF = 1.089\ (D)^2$ (For computing BAF of prism with known diopter strength)
D = Diopters

3. *Miscellaneous*

(a) $D = \sqrt{\dfrac{BAF}{1.089}}$ D = Diopters

(b) $D = \dfrac{100 \times w}{d}$

(c) $w = \dfrac{d \times D}{100}$ (Must know diopter strength first)

(d) $d = \dfrac{100 \times w}{D}$

(e) $d = w\sqrt{\dfrac{10{,}890}{BAF}}$

(f) $w = \sqrt{\frac{d^2 \times BAF}{10{,}890}}$ (To determine width of target when using prism or gauge to measure a distance such as plot radius)

(g) Plot radius factor $= \frac{d}{12}$

(h) Crossarm factor $= \frac{1}{d}$ (For constructing angle-gauge)
(Must be in same units)

4. *Determination of ratio for a predetermined factor*

Assuming a 15 factor gauge is to be constructed:
What is the ratio? What is the crossarm factor?

$$BAF = 10{,}890 \left(\frac{w}{d}\right)^2$$

$$15 = 10{,}890 \left(\frac{w}{d}\right)^2 \qquad \text{(Assume one foot for w)}$$

$$15 = 10{,}890 \times \frac{1^2}{d^2}$$

$$15 = \frac{10{,}890}{d^2}$$

$$d^2 = \frac{10{,}890}{15}$$

$$d = \sqrt{726}$$

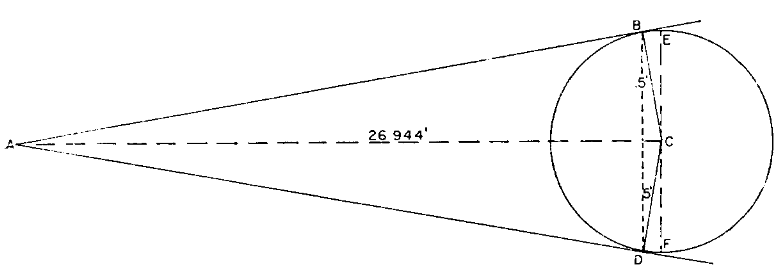
$$d = 26.944$$

Ratio, therefore, is 1 : 26.944 and crossarm factor is $\frac{1}{26.944}$ or .03711. (Note that crossarm factor is the sine of the angle for a particular factor.)

5. *Determination of angle for a given basal area factor*

Right triangle solution*

a. Assume a BAF of 15 and tree diameter or target width of one foot.



$$\angle BAD = \angle BAC + \angle CAD$$

$$\text{Sine BAD} = 2 \times \frac{.5}{26.944}$$

$$\text{Sine BAD} = 2 \times .0185567$$

$$\text{Sine BAD} = .03711$$

$$\angle BAD = 127.60 \text{ minutes}$$

b. Assuming target width of one foot.

From above:

$$\text{Sine BAD} = 2 \times \frac{.5}{d}$$

$$d = w \sqrt{\frac{10{,}890}{BAF}}$$

$$\text{Sine BAD} = 2 \times \frac{.5}{1 \sqrt{\frac{10{,}890}{BAF}}}$$

$$= 2 \times \frac{.5}{\sqrt{\frac{10{,}890}{BAF}}}$$

$$= \frac{1}{\sqrt{\frac{10{,}890}{BAF}}}$$

$$= \sqrt{\frac{BAF}{10{,}890}}$$

$$= \sqrt{\frac{15}{10{,}890}}$$

$$= \sqrt{.00137741}$$

$$= .03711$$

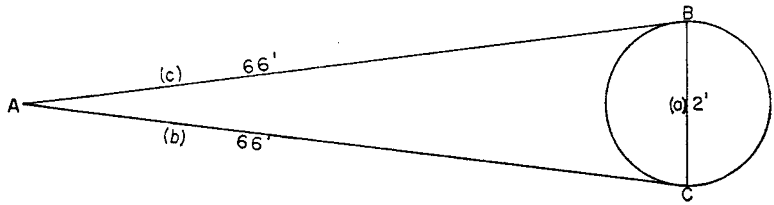
$$\angle BAD = 127.60 \text{ minutes}$$

---

* The observer actually sees line BD rather than EF; however the difference is negligible for all practical purposes. As the angle decreases this difference becomes infinitesimal.

Oblique triangle solution

Assuming triangle BAC to be an oblique triangle and BA or CA distance to tree center. BAF 10 with ratio of 1 : 33.



$$\text{Sine } \tfrac{1}{2} A = \sqrt{\frac{(S-b)\ (S-c)}{bc}} \qquad [S = \tfrac{1}{2}(a + b + c)]$$

$$= \sqrt{\frac{(67-66)\ (67-66)}{66 \times 66}}$$

$$= \sqrt{\frac{1 \times 1}{4356}}$$

$$= \sqrt{.000229568}$$

$$\text{Sine } A = 2 \times \sqrt{.000229568}$$

$$= 2 \times .0151515$$

$$= .030303$$

$$\angle BAC = 104.18 \text{ minutes}$$

(For angle of any factor: Sine of angle $= .00958264 \times \sqrt{BAF}$)

TABLE 5

Size of Plots and Conversion Factors for Trees of Various Diameter Classes

BAF 5

DBH (Inches)	Plot Radius (Feet) (DBH x 3.889)	Plot Size (Acres)	Per Acre* Conversion Factor	Basal Area of One Tree (Sq. Ft.)
4	15.56	.0174	57.30	.087
6	23.33	.0392	25.47	.196
8	31.11	.0698	14.33	.349
10	38.89	.1091	9.17	.545
12	46.67	.1571	6.37	.785
14	54.45	.2138	4.68	1.069
16	62.22	.2792	3.58	1.396
18	70.00	.3534	2.83	1.767
20	77.78	.4363	2.29	2.182
22	85.56	.5280	1.89	2.640
24	93.34	.6283	1.59	3.142

BAF 10

DBH (Inches)	Plot Radius (Feet) (DBH x 2.75)	Plot Size (Acres)	Per Acre* Conversion Factor	Basal Area of One Tree (Sq. Ft.)
4	11.00	.0087	114.62	.087
6	16.50	.0196	50.94	.196
8	22.00	.0349	28.65	.349
10	27.50	.0545	18.34	.545
12	33.00	.0785	12.74	.785
14	38.50	.1069	9.36	1.069
16	44.00	.1396	7.16	1.396
18	49.50	.1767	5.66	1.767
20	55.00	.2182	4.59	2.182
22	60.50	.2640	3.79	2.640
24	66.00	.3142	3.18	3.142
26	71.50	.3687	2.71	3.687
28	77.00	.4276	2.34	4.276
30	82.50	.4909	2.04	4.909
32	88.00	.5585	1.79	5.585
34	93.50	.6305	1.59	6.305
36	99.00	.7069	1.42	7.069

TABLE 5 (cont.)

BAF 20

DBH (Inches)	Plot Radius (Feet) (DBH x 1.9445)	Plot Size (Acres)	Per Acre* Conversion Factor	Basal Area of One Tree (Sq. Ft.)
4	7.78	.0044	229.29	.087
6	11.67	.0098	101.91	.196
8	15.56	.0175	57.32	.349
10	19.44	.0273	36.69	.545
12	23.33	.0393	25.48	.785
14	27.22	.0534	18.72	1.069
16	31.11	.0698	14.33	1.396
18	35.00	.0884	11.32	1.767
20	38.89	.1091	9.17	2.182
22	42.78	.1320	7.58	2.640
24	46.67	.1571	6.37	3.142
26	50.56	.1844	5.43	3.687
28	54.45	.2138	4.68	4.276
30	58.34	.2454	4.08	4.909
32	62.22	.2792	3.58	5.585
34	66.11	.3152	3.17	6.305
36	70.00	.3534	2.83	7.069
38	73.89	.3938	2.54	7.876
40	77.78	.4363	2.29	8.727
42	81.67	.4810	2.08	9.621
44	85.56	.5280	1.90	10.559
46	89.45	.5770	1.73	11.541
48	93.34	.6283	1.59	12.566
50	97.23	.6818	1.47	13.635
52	101.11	.7374	1.36	14.748

*Number of trees per acre that each tallied tree represents. (This times basal area of respective diameter equals basal area factor.)

Note how unusually small the "plot" area can be when using a large basal area factor in small timber or how unusually large the "plot" area can be when using a small basal area factor in large timber. Choose the factor most suitable to the condition.

TABLE 6

Slope Correction

(Correction factors for basal area or volume per acre for tallies taken on a slope.)

Maximum Per Cent of Slope at Sampling Point	Maximum Degrees of Slope at Sampling Point	Multiply Tree Count By:
15	8° 32′	1.01
20	11° 19′	1.02
25	14° 2′	1.03
30	16° 42′	1.04
35	19° 17′	1.06
40	21° 48′	1.08
45	24° 14′	1.10
50	26° 34′	1.12
55	28° 49′	1.14
60	30° 58′	1.17
65	33° 1′	1.19
70	35° 0′	1.22
75	36° 52′	1.25
80	38° 40′	1.28
85	40° 22′	1.31
90	41° 59′	1.34
95	43° 32′	1.38
100	45° 0′	1.41

Slopes less than 15% or approximately 8 degrees are insignificant.

This correction for tree count, basal area and volume per acre is true *only* when it is assumed that in this sampling procedure a proportionate number of trees will be added when the slope distance is corrected to the true horizontal distance. For all practical purposes, however, tree count, basal area and volume can be corrected by using the above multiplier.

TABLE 7

Cutting Guide for Northern Hardwoods

Desirable Stocking per Acre for Good Continuous Growth (From Arbogast, 1957)

DBH Inches	Desirable Stand After Cutting: Trees		Desirable Stand After Cutting: Basal Area	
	Number		Square Feet	
2	118		2.6	
3	53	202	2.6	8
4	31		2.7	
5	21		2.9	
6	15		2.9	
7	12	65	3.2	16
8	9		3.1	
9	8		3.5	
10	7		3.8	
11	6		4.0	
12	5	28	3.9	22
13	5		4.6	
14	5		5.3	
15	4		4.9	
16	4		5.6	
17	3	17	4.7	26
18	3		5.3	
19	3		5.9	
20	2		4.4	
21	2		4.8	
22	2	8	5.3	20
23	1		2.9	
24	1		3.1	
Total	320		92	

TABLE 8

Basal Area Spacing Guide

(From U. S. Forest Service Timber Management Field Book, Region 9)
Square Feet of Basal Area per Acre

DBH Inches	Square Feet of Basal Area per Acre: 40	50	60	70	80	90	100	110	120	140
	Square Spacing to Nearest Foot									
2	5	4	4	4	3	3	3	3	3	3
3	7	7	6	6	5	5	5	4	4	4
4	10	9	8	7	7	6	6	6	6	5
5	12	11	10	9	9	8	8	7	7	7
6	15	13	12	11	10	10	9	9	8	8
7	17	15	14	13	12	11	11	10	10	9
8	19	17	16	15	14	13	12	12	11	10
9	22	20	18	17	16	15	14	13	13	11
10	24	22	20	18	17	16	15	15	14	13
12	29	26	24	21	21	19	18	18	17	16
14	34	31	28	26	24	23	22	21	20	18
16	39	35	32	29	28	26	25	24	24	21
18	44	39	36	33	31	29	28	26	25	23
20	49	44	40	37	34	32	31	29	28	26

TABLE 9

Cordwood and Board Foot Volumes per Square Foot of Basal Area

Cord Volumes per Square Foot of Basal Area*

Tree DBH	No. of 8-Foot Sticks 1	2	3	4	5	6	7	Tree Basal Area
5	.081	.140	—	—	—	—	—	.136
6	.087	.143	.204	—	—	—	—	.196
7	.086	.142	.198	.255	—	—	—	.267
8	.089	.143	.195	.249	.304	—	—	.349
9	.090	.147	.199	.247	.294	.346	—	.442
10	.090	.150	.204	.244	.294	.345	.387	.545
11	.091	.152	.208	.250	.288	.335	.379	.660
12	—	.154	.210	.252	.287	.331	.382	.785
13	—	—	.214	.256	.291	.331	.380	.922
14	—	—	—	.255	.291	.330	.374	1.069
15	—	—	—	—	.297	.334	.375	1.227
Average	.088	.146	.204	.251	.293	.336	.380	

Board Foot Volumes per Square Foot of Basal Area Scribner Decimal C*

Tree DBH	No. of 16-Foot Logs ½	1	1½	2	2½	3	4	Tree Basal Area
10	31	55	—	—	—	—	—	.545
12	36	61	84	99	—	—	—	.785
14	37	65	90	108	132	—	—	1.069
16	—	67	92	113	137	160	—	1.396
18	—	69	95	117	140	165	201	1.767
20	—	71	97	120	145	168	206	2.182
22	—	—	99	124	148	170	212	2.640
24	—	—	102	127	150	175	220	3.142
26	—	—	103	130	153	176	222	3.690
28	—	—	—	131	156	178	224	4.28
Rounded Average	35	65	95	119	145	170	214	

Multiply rounded volumes by basal area factor to obtain total volume represented by each respective height class. For example: each 2-log tree tallied on a point with a 10-factor gauge represents 1200 board feet per acre. Each log therefore represents approximately 600 board feet per acre.

To use 600 board feet per log or .6 cord per stick timber must average 2 to 2½ logs and 4 to 5 sticks in height respectively.

*Composite Volume Tables, Tables 1 and 6, U.S.D.A. Technical Bulletin No. 1104.

TABLE 10

Cordwood and Board Foot Conversion Factors for Various Basal Area Factors in the Lake States

(Volumes represented by each 16' log or 8' stick tallied)

Basal Area Factor	Board Feet (Scribner Decimal C)	Cords
3.5	210	.21
4.0	240	.24
4.5	270	.27
5.0	300	.30
5.5	330	.33
6.0	360	.36
6.5	390	.39
7.0	420	.42
7.5	450	.45
8.0	480	.48
8.5	510	.51
9.0	540	.54
9.5	570	.57
10.0	600	.60
10.5	630	.63
11.0	660	.66
11.5	690	.69
12.0	720	.72
12.5	750	.75
13.0	780	.78
13.5	810	.81
14.0	840	.84
14.5	870	.87
15.0	900	.90
15.5	930	.93
16.0	960	.96
16.5	990	.99
17.0	1020	1.02
17.5	1050	1.05
18.0	1080	1.08
18.5	1110	1.11
19.0	1140	1.14
19.5	1170	1.17
20.0	1200	1.20
20.5	1230	1.23
21.0	1260	1.26
21.5	1290	1.29
22.0	1320	1.32
22.5	1350	1.35
23.0	1380	1.38
23.5	1410	1.41
24.0	1440	1.44
24.5	1470	1.47
25.0	1500	1.50

Figures based on 60 board feet per log and .06 cord per stick per square foot of basal area for trees of average heights.

TABLE 11

Number of 10-Factor Sample Points Required

(From U. S. Forest Service Timber Management Field Book, Region 9)

Type of Stand	Limits of Error in Per Cent 19 Times Out of 20					Coeff. of Variation in Per Cent
	8	10	12	15	30	
	Required Number of Points					
Sawtimber Volume Only						
Poorly Stocked	980	625	430	280	70	125
Medium Stocked	230	150	100	70	20	60
Well Stocked	230	150	100	70	20	60
Sawtimber and Cordwood Volume						
Poorly Stocked	620	400	280	180	45	100
Medium Stocked	230	150	100	70	20	60
Well Stocked	160	100	70	50	15	50
Cordwood Volume Only						
2 Cd. Per Acre	900	580	400	260	60	121
4 Cd. Per Acre	580	370	260	160	40	96
6 Cd. Per Acre	420	270	190	120	30	82
8 Cd. Per Acre	310	200	140	90	20	71
10 Cd. Per Acre	250	160	110	70	20	63
12 Cd. Per Acre	200	130	90	60	15	57
14 Cd. Per Acre	170	110	80	50	15	52
16 Cd. Per Acre	140	90	60	40	10	47
18 Cd. Per Acre	120	80	50	40	10	44
20 Cd. Per Acre	90	60	40	30	10	40

TABLE 12

The Intensity of Point Sampling (Factor 10) in Relation to Mean Volume and the Accuracy Desired

(From U. S. Forest Service Timber Management Field Book, Region 9)

Mean Vol. Per Acre (Cords)	Coef. of Var. (c)	Number of Points (n) Needed[1] For One Standard Error									
		2½%	5%	7½%	10%	15%	20%	25%	30%	35%	40%
1.0	1.50				225	100	56	36	25	18	14
2.0	1.21			260	146	65	37	23	16	12	9
3.0	1.06			200	112	50	28	18	12	9	
4.0	.96		370	164	92	41	23	15	10		
5.0	.88		310	138	77	34	19	12	9		
6.0	.82		269	119	67	30	17	11			
7.0	.76		231	103	58	26	14	9			
8.0	.71		202	90	50	22	13	8			
9.0	.68		185	82	46	21	12	7			
10.0	.63		159	71	40	18	10	6			
12.0	.57		130	58	32	14	8				
14.0	.52	433	108	48	27	12	7				
16.0	.47	353	88	39	22	10	6				
18.0	.44	311	78	34	19	9	5				
20.0	.40	256	64	28	16	7	4				

[1] $n = \frac{c^2}{e^2}$

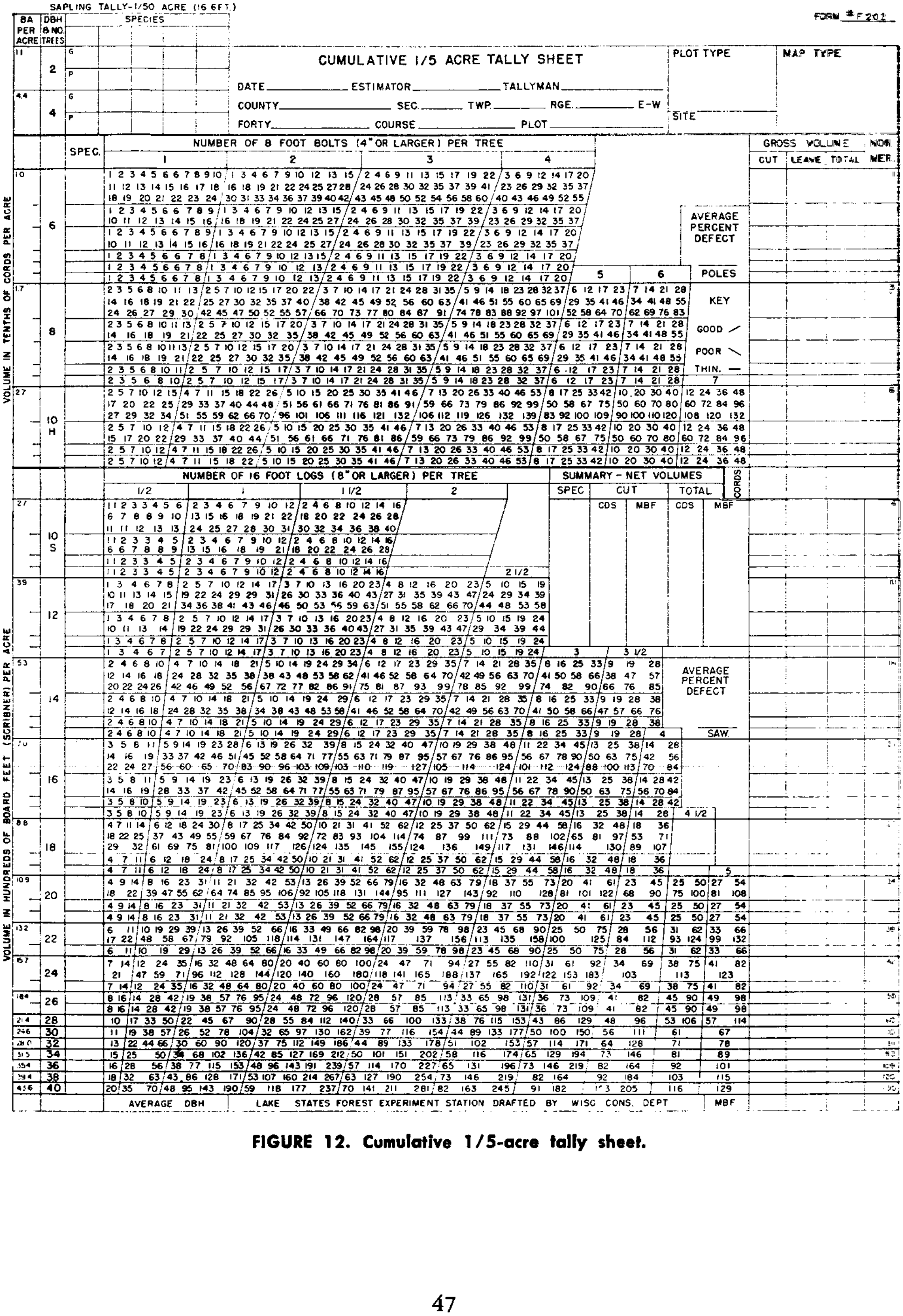


FIGURE 12. Cumulative 1/5-acre tally sheet.

Block or Unit____________ Course____________ Plot____________ Type____________

Crew____ Date____________

Forty____________ Section____________ Twp.____________ Range____________

DBH	Species	Basal Area sq. ft.															B.A. per acre		
4	____	1	2	3	3	4	5	6	7	8	9	10	10	11	12	13			
	____	1	2	3	3	4	5	6	7	8	9	10	10	11	12	13			
	____	1	2	3	3	4	5	6	7	8	9	10	10	11	12	13			
5	____	1	3	4	5	7	8	10	11	12	14	15	16	18	19				
	____	1	3	4	5	7	8	10	11	12	14	15	16	18	19				
	____	1	3	4	5	7	8	10	11	12	14	15	16	18	19				
6	____	2	4	6	8	10	12	14	16	18	20	22	24	26	27				
	____	2	4	6	8	10	12	14	16	18	20	22	24	26	27				
	____	2	4	6	8	10	12	14	16	18	20	22	24	26	27				
7	____	3	5	8	11	13	16	19	21	24	27	29	32	35	37				
	____	3	5	8	11	13	16	19	21	24	27	29	32	35	37				
	____	3	5	8	11	13	16	19	21	24	27	29	32	35	37				
8	____	3	7	10	14	17	21	24	28	31	35	38	42	45	49				
	____	3	7	10	14	17	21	24	28	31	35	38	42	45	49				
9	____	4	9	13	18	22	26	31	35	40	44	49	53	57	62				
	____	4	9	13	18	22	26	31	35	40	44	49	53	57	62				
10	____	5	11	16	22	27	33	38	44	49	54	60	65						
	____	5	11	16	22	27	33	38	44	49	54	60	65						
11	____	7	13	20	26	33	40	46	53	59	66	73	79						
	____	7	13	20	26	33	40	46	53	59	66	73	79						
12	____	8	16	24	31	39	47	55	63	71	78	86							
	____	8	16	24	31	39	47	55	63	71	78	86							
13	____	9	18	28	37	46	55	64	74	83	92	101							
	____	9	18	28	37	46	55	64	74	83	92	101							
14	____	11	21	32	43	53	64	75	85	96	107	118							
	____	11	21	32	43	53	64	75	85	96	107	118							
15	____	12	25	37	49	61	74	86	98	110	123	135							
	____	12	25	37	49	61	74	86	98	110	123	135							
16	____	14	28	42	56	70	84	98	112	126	140								
	____	14	28	42	56	70	84	98	112	126	140								
17	____	16	32	47	63	79	95	110	126	142	158								
	____	16	32	47	63	79	95	110	126	142	158								
18	____	18	35	53	71	88	106	124	141	159	177								
	____	18	35	53	71	88	106	124	141	159	177								
19	____	20	39	59	79	98	118	138	157	177	197								
	____	20	39	59	79	98	118	138	157	177	197								
20	____	22	44	65	87	109	131	153	174	196	218								
	____	22	44	65	87	109	131	153	174	196	218								
21	____	24	48	72	96	120	144	168	192										
	____	24	48	72	96	120	144	168	192										
22	____	26	53	79	106	132	158	185	211										
	____	26	53	79	106	132	158	185	211										
23	____	29	58	86	115	144	173	202	231										
	____	29	58	86	115	144	173	202	231										

FIGURE 13. Cumulative 1/10 basal area tally sheet.

STAND EXAMINATION TALLY SHEET
FORM 2400-32

DEPARTMENT OF NATURAL RESOURCES

FOREST			COMP. NO.	STAND NO.
SECTION	T.	R.	DESC.	MAP TYPE
ESTIMATOR			DATE	NO. OF POINTS

Point No.	SPECIES							Total Cords	BASAL AREA Sap	Pole	Saw	Total
1.												
2.												
3.												
4.												
5.												
6.												
7.												
8.												
9.												
10.												
Total												
Per Acre												

SMALL SAWTIMBER

Sp.	No. of 16–foot logs BAF 10: ½	1	1½	2	2½	3	D.B.H. Range	BD. FT.	GRTH.	AGE	HT.
	3 7 10 13 17	6 12 18 24 30 36 42 48 54 60 66 72 78 84	9 17 26 35 43 52 61 70 78 87 96 104 113	10 21 32 42 52 63 73 84 95 105 115 126	13 26 39 52 65 78	15 30 45 60					
	3 7 10 13 17	6 12 18 24 30 36 42 48 54 60 66 72 78 84	9 17 26 35 43 52 61 70 78 87 96 104 113	10 21 32 42 52 63 73 84 95 105 115 126	13 26 39 52 65 78	15 30 45 60					
	3 7 10	6 12 18 24 30 36 42	9 17 26 35 43 52 61	10 21 32 42 52 63 73	13 26 39	15 30	AV. D.B.H.				
	3 7 10	6 12 18 24 30 36 42	9 17 26 35 43 52 61	10 21 32 42 52 63 73	13 26 39	15 30					
	3 7 10	6 12 18 24 30 36 42	9 17 26 35 43 52 61	10 21 32 42 52 63 73	13 26 39	15 30					
	3 7 10	6 12 18 24 30 36 42	9 17 26 35 43 52 61	10 21 32 42 52 63 73	13 26 39	15 30					
	3 7 10	6 12 18 24 30 36 42	9 17 26 35 43 52 61	10 21 32 42 52 63 73	13 26 39	15 30					

LARGE SAWTIMBER

Sp.	½	1	1½	2	2½	3	3½	4	4½	5
	4 8 12 16	7 14 20 27 34 41	9 19 29 38 47 57 67 77 85 95 105	12 25 37 49 62 74 86 98 111 123 135	15 29 44 59 74 88 103 118 132	17 34 51 68 85 102 119	19 38 58 77	21 43 65 86	24 48	26 51
	4 8 12 16	7 14 20 27 34 41	9 19 29 38 47 57 67 77 85 95 105	12 25 37 49 62 74 86 98 111 123 135	15 29 44 59 74 88 103 118 132	17 34 51 68 85 102 119	19 38 58 77	21 43 65 86	24 48	26 51
	4 8	7 14 20	9 19 29 38 47 57	12 25 37 49 62 74	15 29 44 59 74	17 34 51 68	19 38	21 43	24	26
	4 8	7 14 20	9 19 29 38 47 57	12 25 37 49 62 74	15 29 44 59 74	17 34 51 68	19 38	21 43	24	26
	4 8	7 14 20	9 19 29 38 47 57	12 25 37 49 62 74	15 29 44 59 74	17 34 51 68	19 38	21 43	24	26

Remarks:

	BD. FT.	GRTH.	AGE	HT.
Total				
Av. Per Acre				

REV. 1-70

FIGURE 14. Stand examination tally sheet.

Cumulative Volume Tally Sheet for Woodland Analysis

Point Sampling Basal Area Factor of 10

Owner ______________ Date ______________ Comp. ______________

Estimator ______________ County or Working Circle ______________

No. Points Tallied	Species Legend Percent of Soundness	Stand. No. ______ Stand. Age ______ Site Index ______	T. ____ R. ____ Sec. ____ 40's ______ Timber Type ______ Acres ______

		ROUGH CORDS (8 FOOT STICKS TO 4 INCH TOP D.I.B.)																																	CULL TREES	TOTALS
		1 STICK				2 STICKS							3 STICKS						RINGS PER						DBH		RINGS		DBH	RINGS		DBH	RINGS			
GROWING STOCK	2																		LAST INCH																	
	4																		4 STICKS																	
	6	9	17	26	35	14	29	43	57	71	86	100	20	41	61	82	102	122	24	48	72	96	120	144												
		43	52	61	69	114	129	143	157	171	186	200	143	163	184	204	224	245	168	192	216	240	264	287	5 STICKS				6 STICKS							
	8	9	18	27	36	14	29	43	57	72	86	100	20	39	59	78	97	117	25	50	75	100	125	150	30	61	91	122	33	67	100					
		44	53	62	71	115	129	143	158	172	187	201	136	156	175	195	214	234	175	200	224	249	274	299	152	182	213	243	133	166	199	7 STICKS				
	10	9	18	27	36	15	30	45	60	75	90	105	20	41	61	81	102	122	24	49	73	98	122	146	29	59	88	117	34	69	103	39	77	116		
		45	54	63	72	120	135	150	165	180	195	210	142	163	183	204	224	244	171	195	219	244	268	293	147	176	205	235	138	172	207	155	193	232		
HARVEST STOCK	6	9	17	26	35	14	29	43	57	71	86	100	20	41	61	82	102	122	24	48	72	96	120	144	REPRODUCTION SPECIES											
		43	52	61	69	114	129	143	157	171	186	200	143	163	184	204	224	245	168	192	216	240	264	287	1/250 Ac (7.4' RADIUS)											
	8	9	18	27	36	14	29	43	57	72	86	100	20	39	59	78	97	117	25	50	75	100	125	150	30	61	91	122	33	67	100					
		44	53	62	71	115	129	143	158	172	187	201	136	156	175	195	214	234	175	200	224	249	274	299	152	182	213	243	133	166	199					
	10	9	18	27	36	15	30	45	60	75	90	105	20	41	61	81	102	122	24	49	73	98	122	146	29	59	88	117	34	69	103	39	77	116		
		45	54	63	72	120	135	150	165	180	195	210	142	163	183	204	224	244	171	195	219	244	268	293	147	176	205	235	138	172	207	155	193	232		
	12	9	18	27	36	15	31	46	62	77	92	108	21	42	63	84	105	126	25	50	76	101	126	151	29	57	80	115	33	66	99	38	76	115		
		45	54	62	71	123	139	154	169	185	200	216	147	168	189	210	231	252	177	202	227	252	277	301	143	172	201	229	132	166	199	153	191	229		
	14	9	18	27	36	16	31	47	62	78	94	109	21	43	64	85	107	128	26	51	77	102	128	153	29	58	87	116	33	66	99	37	75	112		
		45	54	63	72	125	141	156	172	188	203	218	149	171	192	213	235	256	179	205	231	257	282	308	145	175	204	233	132	166	199	150	188	226		
	16	9	18	26	35	16	31	47	63	79	95	110	21	43	65	86	108	129	26	52	79	105	132	158	30	60	90	120	34	67	101	38	76	114		
		44	53	61	70	126	142	158	174	190	206	222	151	173	194	216	238	259	184	211	238	264	291	317	150	180	211	241	135	169	203	158	191	229		

	D B H	BOARD FEET (16 FOOT LOGS - VARIABLE TOP TO 8 INCH D.I.B.)																																	CULL TREES
		½ LOG				1 LOG							1½ LOGS																						
GROWING STOCK	10	3	6	9	12	5	11	16	22	27	33	38	7	15	22	29	37	44	2 LOGS						SCRIBNER D.C. RULE										
	12	4	7	11	14	6	12	18	24	30	37	43	8	17	25	34	42	50	10	20	30	40	50	59											
		18	21	25	28	49	55	61	67	73	79	85	59	67	76	84	92	101	69	79	89	99	109	119	2½ LOGS				3 LOGS			3½ LOGS			
	14	4	7	11	15	6	13	19	26	33	39	45	9	18	27	36	45	54	11	22	32	43	54	65	13	26	40	53	15	30	45	16	32	48	
		19	23	26	30	52	59	66	72	79	85	92	63	72	81	90	99	108	76	87	98	109	120	131	66	79	92	106	60	75	90	64	80	96	
	16	4	8	12	16	7	13	20	27	33	40	47	9	19	28	37	47	56	11	23	34	45	56	68	14	27	41	55	16	32	48	18	36	53	
	18	4	8	12	16	7	14	21	28	35	41	48	10	19	28	38	48	57	12	23	35	47	58	70	14	28	42	56	17	33	49	18	37	55	
	20	4	8	12	17	7	14	21	28	36	43	50	10	19	29	39	48	58	12	24	36	48	60	72	14	29	43	58	17	34	50	19	38	57	
	22	4	8	13	17	7	15	22	29	37	44	52	10	20	30	40	49	59	12	25	37	50	62	74	15	30	44	59	17	34	51	19	39	58	
	24	4	9	13	17	7	15	22	30	38	45	53	10	20	30	40	50	61	13	25	38	51	63	76	15	30	45	60	17	35	53	20	39	59	
	26	5	9	13	18	8	15	23	30	38	46	53	10	21	31	41	51	63	13	26	39	52	65	78	15	31	46	61	18	35	53	20	40	60	
	28	5	9	14	18	8	15	23	31	38	46	53	11	21	31	42	52	63	13	26	39	52	65	79	16	31	47	63	18	36	53	20	40	60	
	30+	5	9	14	18	8	16	23	31	38	46	54	11	21	32	42	52	63	13	26	40	52	65	78	16	31	47	63	18	36	53	20	40	60	
HARVEST STOCK	12	4	7	11	14	6	12	18	24	30	37	43	8	17	25	34	42	50	10	20	30	40	50	59											
	14	4	7	11	15	6	13	19	26	33	39	45	9	18	27	36	45	54	11	22	32	43	54	65	13	26	40	53	15	30	45	16	32	48	
	16	4	8	12	16	7	13	20	27	33	40	47	9	19	28	37	47	56	11	23	34	45	56	68	14	27	41	55	16	32	48	18	36	53	
	18	4	8	12	16	7	14	21	28	35	41	48	10	19	28	38	48	57	12	23	35	47	58	70	14	28	42	56	17	33	49	18	37	55	
	20	4	8	12	17	7	14	21	28	36	43	50	10	19	29	39	48	58	12	24	36	48	60	72	14	29	43	58	17	34	50	19	38	57	
	22	4	8	13	17	7	15	22	29	37	44	52	10	20	30	40	49	59	12	25	37	50	62	74	15	30	44	59	17	34	51	19	39	58	
	24	4	9	13	17	7	15	22	30	38	45	53	10	20	30	40	50	61	13	25	38	51	63	76	15	30	45	60	17	35	53	20	39	59	
	26	5	9	13	18	8	15	23	30	38	46	53	10	21	31	41	51	62	13	26	39	52	65	78	15	31	46	61	18	35	53	20	40	60	
	28	5	9	14	18	8	15	23	31	38	46	53	11	21	31	42	52	63	13	26	39	52	65	79	16	31	47	63	18	36	53	20	40	60	
	30	5	9	14	18	8	16	23	31	38	46	54	11	21	32	42	52	63	13	26	40	52	65	78	16	31	47	63	18	36	53	20	40	60	
	32	5	9	14	19	8	16	24	32	40	47	55	11	22	32	43	54	65	13	27	40	53	66	80	16	32	48	64	18	37	55	21	41	62	
	34+	5	9	14	19	8	16	24	32	40	48	56	11	22	33	43	54	65	13	27	40	57	67	81	16	32	48	65	19	37	56	21	42	62	

1- Dot tally the number of points, number of culltrees, and 2 and 4 inch D.B.H. trees using species symbol.

2- Board feet per acre - multiply the sum of the last figure tallied in each block by 100, then divide by number of points sampled.

3- Cords per acre - multiply the sum of the last figure tallied in each block by 0.1, then divide by number of points sampled.

Based on point-sampling with a critical angle of 104.18 minutes (basal area factor of 10) and composite volume tables developed at the Lake States Forest Experiment Station, 1955. (Table 1 and Table 6, USDA Tech. Bulletin No. 1104)

(Agriculture - Republic Litho Co., Milwaukee, Wis., 1960)

FIGURE 15. Cumulative volume tally sheet for woodland analysis (point-sampling basal area factor of 10).

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